

**Estudio Analítico - Gráfico**  
**de los**  
**Poliedros Regulares**  
**y de sus**  
**Derivados y Conjugados**

LAMINAS 16 AL 25

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Prof. T. Alvarez Peralto



R. 7815  
119025087Lámina 16

## ENUNCIADO

Representar por el método gráfico-analítico, en los planos I, II y III, el icosaedro conjugado de un dodecaedro regular de 35.7 mm de lado, obtenido aquel al trazar por los puntos medios de las aristas del dodecaedro dado, rectas perpendiculares al plano determinado por cada una de sus aristas y el centro O del mismo, cuyas coordenadas son:  $O(72, 72, 85)$  mm.

Calcular previamente sus cotas fundamentales que servirán de comprobación al trazado gráfico.

Dibujar en formato A3V y a escala 1:1.

DATOS  $O(72, 72, 85)$  mm

$l_2 = 35.7$  mm

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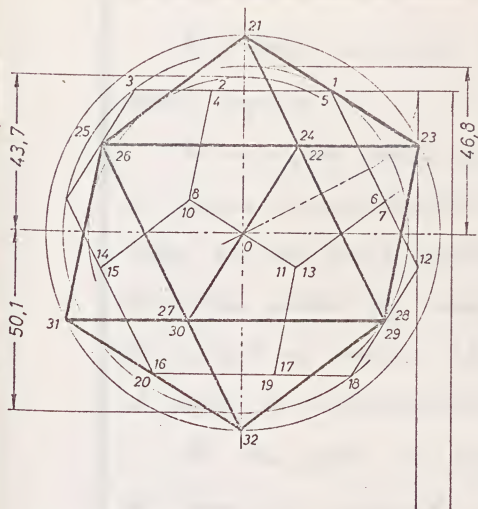




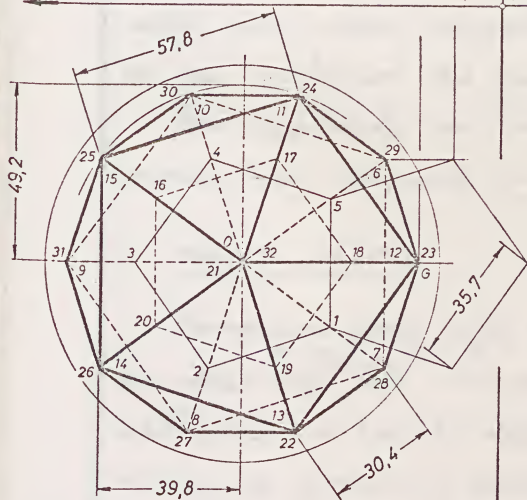




I



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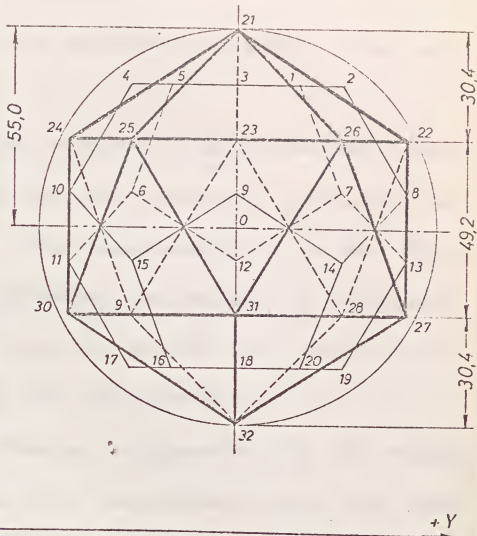
### NUMERACIÓN DE VÉRTICES

Dodecadro dato..... 1 al 20

Icosaedro conjugado..... 21 al 32

+Y

III



+Y

### ENUNCIADO

Representar por el método gráfico-analítico, en los planos I II y III, el icosaedro conjugado de un dodecaedro regular de 35,7 mm de lado, obtenido aquél al trazar por los puntos medios de las aristas del dodecaedro, rectas perpendiculares al plano determinado por cada una de sus aristas y el centro O del mismo, cuyas coordenadas son: O (72, 72, 85) milímetros.

Calcular previamente sus cotas fundamentales que servirán de comprobación al trazado gráfico.

Dibujar en formato A3v y a escala 1:1.

	Propuesta	De entrega	Entregada	Calificación	(firma)	Escuela
Fecha:						Curso
Alumno:						
Escala	Poliedros regulares convexos conjugados					Lámina 16
1:1						Curso 12 -15

II



El poliedro conjugado de un dodecaedro regular, es un icosaedro regular.

El conjugado pedido con las condiciones del enunciado tiene sus aristas perpendiculares a las del dodecaedro, y ambas se cortan en sus puntos medios. Por consiguiente, la esfera tangente a las aristas de ambos poliedros, es común, y el punto de contacto de dicha esfera con las aristas es precisamente el punto de intersección de las mismas.

Por otra parte, si proyectamos ortogonalmente los vértices del icosaedro conjugado sobre las respectivas caras del dodecaedro dado, dicha proyección coincide con el centro de cada cara pentagonal del dodecaedro.

Estas propiedades nos permiten resolver fácilmente la representación propuesta en el enunciado.

### PROCESO GRÁFICO

Procedamos previamente a la representación del dodecaedro dado <sup>en los planos I, II y III</sup> cuyo lado, conocido, es de 35.7 mm. Para ello utilizaremos el trazado dado en el "Proceso gráfico" de la lámina 4, determinando previamente el valor del radio  $c$  de la esfera inscrita (ver Proceso gráfico-analítico) cuyo valor será:

$$c = 1,1135 \times 35,7 = 39,8 \text{ mm}$$

Seguidamente, completaremos en el plano II la proyección del icosaedro pedido, cuyo contorno, según demostraremos seguidamente, es coincidente con el del dodecaedro dado, por





lo cual la proyección sobre II de dicho icosaedro, es inmediata. Obsérvese que al terminar de dibujar la proyección II, todas las aristas del dodecaedro dado, cortan a las del conjugado en sus puntos medios, ya que esta propiedad en el espacio, se conserva en sus proyecciones.

Dicha propiedad servirá pues de base para obtener la proyección del icosaedro pedido en el plano I.

La proyección del mismo sobre III, se deduce de la I y II.

Con la numeración adecuada de los vértices y centro de ambos poliedros quedará completada la representación pedida.

#### PROCESO GRÁFICO - ANALÍTICO

El cálculo analítico de las magnitudes acotadas en la lámina 16 se desarrolla basándose en el efectuado en la lámina 5, en función del lado  $l'_{20}$  del icosaedro conjugado.

Previamente se determinará el radio  $a'_{20}$  de la esfera inscrita al mismo, teniendo en cuenta la igualdad en ambos poliedros, del radio de la esfera tangente a las aristas.

El radio  $b_{12}$  de la esfera tangente a las aristas del dodecaedro dado, tiene por valor, en función de  $l'_{12}$

$$b_{12} = \frac{3 + \sqrt{5}}{4} l'_{12} = b'_{20}$$

y por otra parte, el lado  $l'_{20}$  del icosaedro conjugado, en función de  $b'_{20}$ , es



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$$l'_{20} = \frac{4}{1+\sqrt{5}} \quad b'_{20} = \frac{4}{1+\sqrt{5}} \times \frac{3+\sqrt{5}}{4} \quad l_{12} = \frac{1+\sqrt{5}}{2} l_{12} \quad [2]$$

conocido  $l'_{20}$  en función de  $l_{12}$ , se calcularán los siguientes valores (ver tabla figura 5):

$$l'_{20} = \frac{1+\sqrt{5}}{2} l_{12} = 1,618034 \times 35,741147 = 57,830393 \quad *$$

siendo la relación  $\frac{l'_{20}}{l_{12}} = \frac{\sqrt{5}+1}{2} = 1,618034 \quad [3]$

$$g \quad a'_{20} = 0,951057 \times 57,830393 = 55,0 \text{ mm}$$

$$b'_{20} = 0,809017 \times \text{id} = 46,8 \text{ "}$$

$$c'_{20} = 0,755761 \times \text{id} = 43,7 \text{ "}$$

$$d'_{20} = 0,577350 \times \text{id} = 33,4 \text{ "}$$

$$f'_{20} = e'_{20} = 0,850651 \times \text{id} = 49,2 \text{ "}$$

$$\angle \psi = 138^{\circ} 11' 23''$$

$$i'_{20} = j'_{20} = 0,525731 \times \text{id} = 30,4 \text{ "}$$

$$h'_{20} = 0,688191 \times \text{id} = 39,8 \text{ "}$$

$$k'_{20} = 0,288675 \times \text{id} = 16,7 \text{ "}$$

\* El valor más aproximado del lado  $l_{12}$  dado, se obtiene del exacto  $a_{20} = 55 \text{ mm}$ ; de donde  $l_{20} = 55 : 0,951057 = 57,830393$  y de aquí;  $b_{20} = 57,830393 \times 0,809017 = 46,785769$ , por lo que será  $l_{12} = 46,785769 : 1,309017 = 35,741147$

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Obsérvese que siendo

$$e'_{20} = \sqrt{\frac{5+\sqrt{5}}{10}} l'_{20} \quad , \quad e_{12} = \sqrt{\frac{5+2\sqrt{5}}{5}} l_{12}$$

y según [3]  $\frac{l'_{20}}{l_{12}} = \frac{\sqrt{5}+1}{2}$ , se verificará que

$$e'_{20} = \sqrt{\frac{5+\sqrt{5}}{10}} l'_{20} = \sqrt{\frac{5+\sqrt{5}}{10}} \times \frac{\sqrt{5}+1}{2} l_{12} = \sqrt{\frac{5+2\sqrt{5}}{5}} l_{12}$$

de donde  $e'_{20} = e_{12}$

El desarrollo del cálculo anterior es el siguiente:

$$\begin{aligned} \sqrt{\frac{5+\sqrt{5}}{10}} \times \frac{\sqrt{5}+1}{2} l_{12} &= \sqrt{\frac{5+\sqrt{5}}{10} \times \frac{(\sqrt{5}+1)^2}{4}} l_{12} = \sqrt{\frac{5+\sqrt{5}}{10} \times \frac{5+1+2\sqrt{5}}{4}} l_{12} = \\ &= \sqrt{\frac{(5+\sqrt{5})(6+2\sqrt{5})}{40}} l_{12} = \sqrt{\frac{(5+\sqrt{5})(3+\sqrt{5})}{20}} l_{12} = \sqrt{\frac{15+3\sqrt{5}+5\sqrt{5}+5}{20}} l_{12} = \\ &= \sqrt{\frac{20+8\sqrt{5}}{20}} l_{12} = \sqrt{\frac{5+2\sqrt{5}}{5}} l_{12} \end{aligned}$$

lo cual nos demuestra que: "El radio de la circunferencia circunscrita al decágono regular, contorno en  $\Pi$  del dodecaedro conjugado, es igual al del dodecaedro dado en la misma proyección."

De aquí se deduce que los lados de ambos decágonos son también iguales, por lo que será:

$$l'_{20} = l_{12}$$



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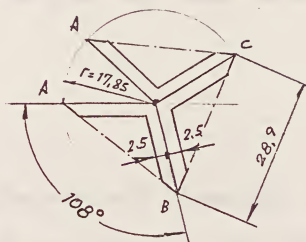
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FIGURA CORPÓREA

El icosaedro conjugado se obtiene por el acoplamiento de 30 triángulos equiláteros de 57,8 mm de lado.

El dodecaedro dado se obtiene formando previamente 20 pirámides triangulares (transparentes y sin base), cuyas caras laterales son 3 triángulos isósceles con base de 28,9 mm (mitad del lado del icosaedro y el ángulo opuesto a dicha base es el inscrito del pentágono regular ( $108^\circ$ ); los lados iguales de este triángulo serán de 17,85 (mitad del lado del dodecaedro dado). Estas pirámides se acoplarán sobre cada una de las veinte caras del conjugado, de forma que sus vértices estén en los puntos medios de las aristas de aquél.

También puede obtenerse el dodecaedro dado, sustituyendo las caras transparentes de las 20 pirámides triangulares, por las obtenidas según el trazado que se representa a continuación.



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Estas 20 pirámides se acoplarán al icosaedro conjugado, colocando sus vértices A, B, C, en los puntos medios de las aristas.

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RESUMEN DEL CÁLCULO DE MAGNITUDES DE ESTE EJERCICIONomenclatura empleado: $l'_{20}$  = Lado del icosaedro conjugado. $d'_{20}$  = Radio de la esfera circunscrita al mismo. $b'_{20}$  = Radio de la esfera tangente a las aristas. $c'_{20}$  = Radio de la esfera inscrita. $d'_{20}$  = Radio de la circunferencia circunscrita al polígono de una cara $\angle \varphi'_{20}$  = Ángulo rectilíneo del diedro formado por dos caras contiguas. $e'_{20}$  = Radio de la circunferencia circunscrita a un pentágono regular de lado  $l'_{20}$ , y también al decágono regular de lado  $i'_{20}$ , contorno de la vista II. $f'_{20}$  = Altura intermedia del contorno de las vistas I y III. $g'_{20}$  = Alturas extremas del contorno de las vistas I y III. $h'_{20}$  = Apotema de un pentágono regular de lado  $l'_{20}$ . $i'_{20}$  = Lado del decágono regular, contorno de la vista II. $k'_{20}$  = Apotema del polígono de una cara. $S'_{20}$  = Superficie. $V'_{20}$  = Volumen.

Los valores anteriores del icosaedro conjugado, pueden obtenerse directamente en función del lado  $l_2$  del dodecaedro dado, ya que el valor conocido de  $l'_{20}$  en función de  $l_{12}$  (ver fórmula [2]), es

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$$l'_{20} = \frac{1+\sqrt{5}}{2} l_{12}$$

Sustituyendo este valor en las fórmulas 43 a 55 de la lámina 5, tendremos:

$$a'_{20} = \frac{\sqrt{10+2\sqrt{5}}}{4} \times \frac{1+\sqrt{5}}{2} l_{12} = \frac{\sqrt{5+2\sqrt{5}}}{2} l_{12}$$

$$\begin{aligned}
 \text{Desarrollo del cálculo anterior: } a'_{20} &= \frac{\sqrt{10+2\sqrt{5}}}{4} \times \frac{1+\sqrt{5}}{2} l_{12} = \\
 &= \frac{\sqrt{(10+2\sqrt{5})(1+\sqrt{5})^2}}{8} l_{12} = \frac{\sqrt{(10+2\sqrt{5})(1+5+2\sqrt{5})}}{8} l_{12} = \frac{\sqrt{(10+2\sqrt{5})(6+2\sqrt{5})}}{8} l_{12} = \\
 &= \frac{\sqrt{2 \times (10+2\sqrt{5})(3+\sqrt{5})}}{8} l_{12} = \frac{\sqrt{2 \times (30+6\sqrt{5}+10\sqrt{5}+10)}}{8} l_{12} = \\
 &= \frac{\sqrt{2 \times (40+16\sqrt{5})}}{8} l_{12} = \frac{\sqrt{2 \times 8 \times (5+2\sqrt{5})}}{8} l_{12} = \frac{4\sqrt{5+2\sqrt{5}}}{8} l_{12} = \\
 &= \boxed{\frac{\sqrt{5+2\sqrt{5}}}{2} l_{12}}
 \end{aligned}$$

$$b'_{20} = \frac{1+\sqrt{5}}{4} \times \frac{1+\sqrt{5}}{2} l_{12} = \frac{1+5+2\sqrt{5}}{8} l_{12} = \frac{3+\sqrt{5}}{4} l_{12}$$

$$c'_6 = \frac{3\sqrt{3} + \sqrt{15}}{12} \times \frac{1+\sqrt{5}}{2} l_{12} = \frac{2\sqrt{3} + \sqrt{15}}{6} l_{12}$$

$$\begin{aligned}
 \text{Desarrollo del cálculo anterior: } c'_6 &= \frac{3\sqrt{3} + \sqrt{15}}{12} \times \frac{1+\sqrt{5}}{2} l_{12} = \\
 &= \frac{3\sqrt{3} + \sqrt{15} + 3\sqrt{15} + 5\sqrt{3}}{24} l_{12} = \frac{8\sqrt{3} + 4\sqrt{15}}{24} l_{12} = \boxed{\frac{2\sqrt{3} + \sqrt{15}}{6} l_{12}}
 \end{aligned}$$

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$$d'_{20} = \frac{\sqrt{3}}{3} \times \frac{1+\sqrt{5}}{2} l_{12} = \frac{\sqrt{3} + \sqrt{15}}{6} l_{12}$$

$$e'_{20} = \sqrt{\frac{5+\sqrt{5}}{10}} \times \frac{1+\sqrt{5}}{2} l_{12} = \sqrt{\frac{5+2\sqrt{5}}{5}} l_{12}$$

Desarrollo del cálculo anterior:  $e'_{20} = \sqrt{\frac{5+\sqrt{5}}{10}} \times \frac{1+\sqrt{5}}{2} l_{12} =$

$$= \sqrt{\frac{(5+\sqrt{5})(1+\sqrt{5})^2}{40}} l_{12} = \sqrt{\frac{(5+\sqrt{5})(6+2\sqrt{5})}{40}} l_{12} = \sqrt{\frac{(5+\sqrt{5})(3+\sqrt{5})}{20}} l_{12} =$$

$$= \sqrt{\frac{15+3\sqrt{5}+5\sqrt{5}+5}{20}} l_{12} = \sqrt{\frac{20+8\sqrt{5}}{20}} l_{12} = \boxed{\sqrt{\frac{5+2\sqrt{5}}{5}} l_{12}}$$

$$f'_{12} = e'_{20} = \sqrt{\frac{5+2\sqrt{5}}{5}} l_{12}$$

$$g'_{20} = \sqrt{\frac{5-\sqrt{5}}{10}} \times \frac{1+\sqrt{5}}{2} l_{12} = \sqrt{\frac{5+\sqrt{5}}{10}} l_{12}$$

Desarrollo del cálculo anterior:  $g'_{20} = \sqrt{\frac{5-\sqrt{5}}{10}} \times \frac{1+\sqrt{5}}{2} l_{12} =$

$$= \sqrt{\frac{(5-\sqrt{5})(1+\sqrt{5})^2}{40}} l_{12} = \sqrt{\frac{(5-\sqrt{5})(6+2\sqrt{5})}{40}} l_{12} = \sqrt{\frac{(5-\sqrt{5})(3+\sqrt{5})}{20}} l_{12} =$$

$$= \sqrt{\frac{15-3\sqrt{5}+5\sqrt{5}-5}{20}} l_{12} = \sqrt{\frac{10+2\sqrt{5}}{20}} l_{12} = \boxed{\sqrt{\frac{5+\sqrt{5}}{10}} l_{12}}$$

$$h'_{20} = \sqrt{\frac{5+2\sqrt{5}}{20}} \times \frac{1+\sqrt{5}}{2} l_{12} = \sqrt{\frac{35+11\sqrt{5}}{40}} l_{12}$$

Desarrollo del cálculo anterior:  $h'_{20} = \sqrt{\frac{5+2\sqrt{5}}{20}} \times \frac{1+\sqrt{5}}{2} l_{12} =$

$$= \sqrt{\frac{(5+2\sqrt{5})(1+\sqrt{5})^2}{80}} l_{12} = \sqrt{\frac{(5+2\sqrt{5})(6+2\sqrt{5})}{80}} l_{12} = \sqrt{\frac{(5+2\sqrt{5})(3+\sqrt{5})}{40}} l_{12} =$$

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$$= \sqrt{\frac{15 + 6\sqrt{5} + 5\sqrt{5} + 10}{40}} l_{12} = \boxed{\sqrt{\frac{25 + 11\sqrt{5}}{40}} l_{12}}$$

$$l'_{20} = g'_{20} = \sqrt{\frac{5 + \sqrt{5}}{10}} l_{12} ;$$

$$k'_{20} = \frac{\sqrt{3}}{6} \times \frac{1 + \sqrt{5}}{2} l_{12} = \frac{\sqrt{3} + \sqrt{15}}{12} l_{12}$$

$$S'_{20} = 5\sqrt{3} \times \left( \frac{1 + \sqrt{5}}{2} l_{12} \right)^2 = \frac{5\sqrt{3} (6 + 2\sqrt{5})}{4} l_{12}^2 = \frac{5\sqrt{3} (3 + \sqrt{5})}{2} l_{12}^2 =$$

$$= \frac{15\sqrt{3} + 5\sqrt{15}}{2} l_{12}^2$$

$$V'_{20} = \frac{15 + 5\sqrt{5}}{12} \times \left( \frac{1 + \sqrt{5}}{2} l_{12} \right)^3 = \frac{5 \cdot (11 + 5\sqrt{5})}{12} l_{12}^3$$

Desarrollo del cálculo anterior:  $V'_{20} = \frac{15 + 5\sqrt{5}}{12} \times \left( \frac{1 + \sqrt{5}}{2} l_{12} \right)^3 =$

$$= \frac{15 + 5\sqrt{5}}{12} \times \frac{1 + 3\sqrt{5} + 15 + 5\sqrt{5}}{8} l_{12}^3 = \frac{15 + 5\sqrt{5}}{12} \times \frac{16 + 8\sqrt{5}}{8} l_{12}^3 =$$

$$= \frac{(15 + 5\sqrt{5})(2 + \sqrt{5})}{12} l_{12}^3 = \frac{30 + 10\sqrt{5} + 15\sqrt{5} + 25}{12} l_{12}^3 = \frac{55 + 25\sqrt{5}}{12} l_{12}^3 =$$

$$= \boxed{\frac{5 \cdot (11 + 5\sqrt{5})}{12} l_{12}^3}$$

Son útiles las siguientes relaciones:

$$a) \frac{l_{12}}{l'_{20}} = \frac{1}{\frac{1 + \sqrt{5}}{2}} = \frac{2}{\sqrt{5} + 1} = \frac{2(\sqrt{5} - 1)}{5 - 1} = \frac{\sqrt{5} - 1}{2} \quad (\text{ver fórmula [2]})$$

Q.1. Find the value of  $\sin^{-1}(\sin \frac{\pi}{6})$

Sol. We know that

$\sin^{-1}(\sin x) = x$  if  $x$  lies in  $[-\frac{\pi}{2}, \frac{\pi}{2}]$

Here  $\frac{\pi}{6}$  lies in  $[-\frac{\pi}{2}, \frac{\pi}{2}]$

$\therefore \sin^{-1}(\sin \frac{\pi}{6}) = \frac{\pi}{6}$

Q.2. Find the value of  $\cos^{-1}(\cos \frac{5\pi}{6})$

Sol. We know that  $\cos^{-1}(\cos x) = x$  if  $x$  lies in  $[0, \pi]$

Here  $\frac{5\pi}{6}$  lies in  $[0, \pi]$

$\therefore \cos^{-1}(\cos \frac{5\pi}{6}) = \frac{5\pi}{6}$

Q.3. Find the value of  $\tan^{-1}(\tan \frac{2\pi}{3})$

Sol. We know that  $\tan^{-1}(\tan x) = x$  if  $x$  lies in  $(-\frac{\pi}{2}, \frac{\pi}{2})$

Here  $\frac{2\pi}{3}$  does not lie in  $(-\frac{\pi}{2}, \frac{\pi}{2})$



$$b) \quad \frac{S_{12}}{S'_{20}} = \frac{\frac{3 \sqrt{25+10\sqrt{5}}}{15\sqrt{3}+5\sqrt{15}} l_{12}^2}{\frac{2}{l_{12}^2}} = \sqrt{\frac{3 \times (5-\sqrt{5})}{10}}$$

Desarrollo del cálculo anterior:

$$\begin{aligned} \frac{S_{12}}{S'_{20}} &= \frac{3 \sqrt{25+10\sqrt{5}}}{15\sqrt{3}+5\sqrt{15}} \frac{l_{12}^2}{\frac{2}{l_{12}^2}} = \\ &= \frac{6 \sqrt{25+10\sqrt{5}}}{15\sqrt{3}+5\sqrt{15}} = \frac{6 \sqrt{25+10\sqrt{5}} \times (15\sqrt{3}-5\sqrt{15})}{675-375} = \frac{2}{6 \times 5 \times \sqrt{25+10\sqrt{5}} \times (3\sqrt{3}-\sqrt{15})} = \\ &= \frac{\sqrt{(25+10\sqrt{5})(3\sqrt{3}-\sqrt{15})^2}}{10} = \frac{\sqrt{(25+10\sqrt{5})(27+15-6\sqrt{45})}}{10} = \\ &= \frac{\sqrt{(25+10\sqrt{5})(42-18\sqrt{5})}}{10} = \frac{\sqrt{6 \times (25+10\sqrt{5})(7-3\sqrt{5})}}{10} = \\ &= \frac{\sqrt{6 \times (175+70\sqrt{5}-75\sqrt{5}-150)}}{10} = \frac{\sqrt{6 \times (25-5\sqrt{5})}}{10} = \frac{\sqrt{6 \times 5 \times (5-\sqrt{5})}}{10} = \\ &= \sqrt{\frac{30(5-\sqrt{5})}{100}} = \sqrt{\frac{3 \times (5-\sqrt{5})}{10}} \end{aligned}$$

$$c) \quad \frac{V_{12}}{V'_{12}} = \frac{\frac{7\sqrt{5}+15}{4} l_{12}^3}{\frac{5 \times (11+5\sqrt{5})}{12} l_{12}^3} = \frac{3 \times (5-\sqrt{5})}{10}$$

Desarrollo del cálculo anterior:

$$\begin{aligned} \frac{V_{12}}{V'_{12}} &= \frac{\frac{7\sqrt{5}+15}{4} l_{12}^3}{\frac{5(11+5\sqrt{5})}{12} l_{12}^3} = \\ &= \frac{12(7\sqrt{5}+15)}{20(11+5\sqrt{5})} = \frac{3(7\sqrt{5}+15)(5\sqrt{5}-11)}{5(125-121)} = \frac{3(175+75\sqrt{5}-77\sqrt{5}-165)}{20} = \\ &= \frac{3 \times (10-2\sqrt{5})}{20} = \boxed{\frac{3 \times (5-\sqrt{5})}{10}} \end{aligned}$$



En el cuadro sinóptico dado a continuación, resumimos  
 estos resultados:

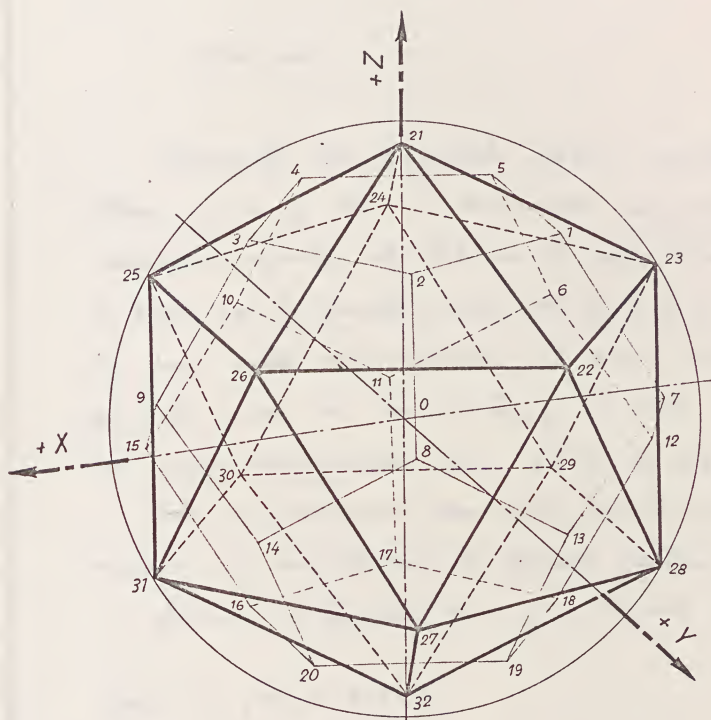
CUADRO SINÓPTICO

Magnitud	Valor exacto	Valor decimal aproximado
(164) $l'_{20}$	$\frac{1+\sqrt{5}}{2} l_{12}$	1, 61 80 34 --- $l_{12}$
(165) $a'_{20}$	$\frac{\sqrt{5+2\sqrt{5}}}{2} l_{12}$	1, 53 88 42 --- $l_{12}$
(166) $b'_{20}$	$\frac{3+\sqrt{5}}{4} l_{12}$	1, 30 90 17 --- $l_{12}$
(167) $c'_{20}$	$\frac{2\sqrt{3}+\sqrt{15}}{6} l_{12}$	1, 22 28 47 --- $l_{12}$
(168) $d'_{20}$	$\frac{\sqrt{3}+\sqrt{15}}{6} l_{12}$	0, 93 41 72 --- $l_{12}$
(169) $2 \varphi_{20}$	$\text{sen } \varphi_{20} = \frac{\sqrt{3}+\sqrt{15}}{6}$	$\text{sen } \varphi_{20} = 0, 93 41 72 \dots$ $2 \varphi_{20} = 138^\circ 11' 22,8''$
(170) $e'_{20}$	$\sqrt{\frac{5+2\sqrt{5}}{5}} l_{12}$	1, 37 63 82 --- $l_{12}$
(171) $f'_{20}$	$\sqrt{\frac{5+2\sqrt{5}}{5}} l_{12}$	1, 37 63 82 --- $l_{12}$
(172) $g'_{20}$	$\sqrt{\frac{5+\sqrt{5}}{10}} l_{12}$	0, 85 06 51 --- $l_{12}$
(173) $h'_{20}$	$\sqrt{\frac{25+11\sqrt{5}}{40}} l_{12}$	1, 11 35 16 --- $l_{12}$
(174) $i'_{20}$	$\sqrt{\frac{5+\sqrt{5}}{10}} l_{12}$	0, 85 06 51 --- $l_{12}$
(175) $k'_{20}$	$\frac{\sqrt{3}+\sqrt{15}}{12} l_{12}$	0, 46 70 86 --- $l_{12}$
(176) $s'_{20}$	$\frac{15\sqrt{3}+5\sqrt{15}}{2} l_{12}^2$	22, 67 28 39 --- $l_{12}^2$
(177) $v'_{20}$	$\frac{5 \cdot (11+5\sqrt{5})}{12} l_{12}^3$	9, 24 18 08 --- $l_{12}^3$
Relaciones entre magnitudes		
(178) $l_{12} : l'_{20}$	$\frac{\sqrt{5}-1}{2}$	0, 61 80 34 ---
(179) $s_{12} : s'_{20}$	$\sqrt{\frac{3 \cdot (5-\sqrt{5})}{10}}$	0, 91 05 93 ---
(180) $v_{12} : v'_{20}$	$\frac{3 \cdot (5-\sqrt{5})}{10}$	0, 82 91 80 ---

$$(s_{12} : s'_{20})^2 = v_{12} : v'_{20}$$

Date		Description		Amount	
1890	Jan 1	Balance		100.00	
	Jan 15	Received from A. B.		50.00	
	Feb 1	Received from C. D.		25.00	
	Feb 15	Received from E. F.		75.00	
	Mar 1	Received from G. H.		100.00	
	Mar 15	Received from I. J.		50.00	
	Apr 1	Received from K. L.		25.00	
	Apr 15	Received from M. N.		75.00	
	May 1	Received from O. P.		100.00	
	May 15	Received from Q. R.		50.00	
	Jun 1	Received from S. T.		25.00	
	Jun 15	Received from U. V.		75.00	
	Jul 1	Received from W. X.		100.00	
	Jul 15	Received from Y. Z.		50.00	
	Aug 1	Received from A. B.		25.00	
	Aug 15	Received from C. D.		75.00	
	Sep 1	Received from E. F.		100.00	
	Sep 15	Received from G. H.		50.00	
	Oct 1	Received from I. J.		25.00	
	Oct 15	Received from K. L.		75.00	
	Nov 1	Received from M. N.		100.00	
	Nov 15	Received from O. P.		50.00	
	Dec 1	Received from Q. R.		25.00	
	Dec 15	Received from S. T.		75.00	
	Total			1000.00	





*Poliedros regulares convexos conjugados*



Figure 1. A cube with internal lines and axes.

Lámina 17

## ENUNCIADO

Representar por el método gráfico-analítico, en los planos I, II y III, el dodecaedro conjugado de un icosaedro regular de 57,8 mm de lado, obtenido aqúel al trazar por los puntos medios de las aristas del icosaedro dado, rectas perpendiculares al plano determinado por cada una de sus aristas y el centro O del mismo, cuyas coordenadas son:  $O(72, 72, 85)$  mm.

Calcular previamente sus cotas fundamentales que servirán de comprobación al trazado gráfico.

Dibujar en formato A3v y a escala 1:1.

DATOS  $O(72, 72, 85)$  mm

$l_{20} = 57,8$  mm



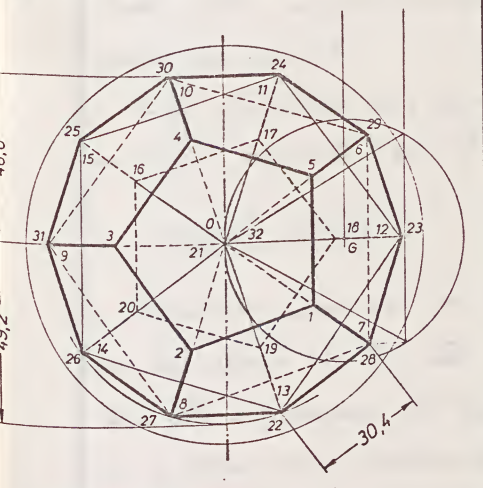
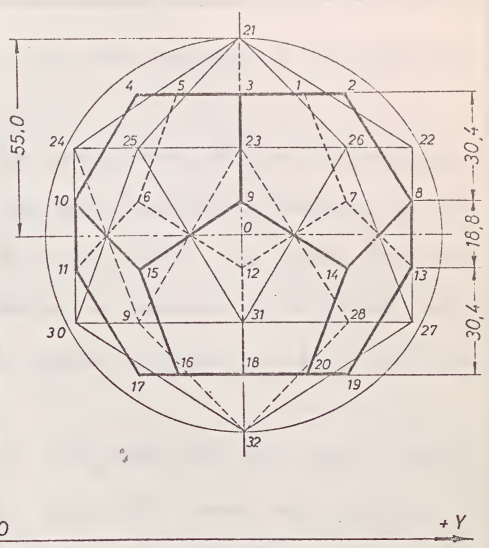
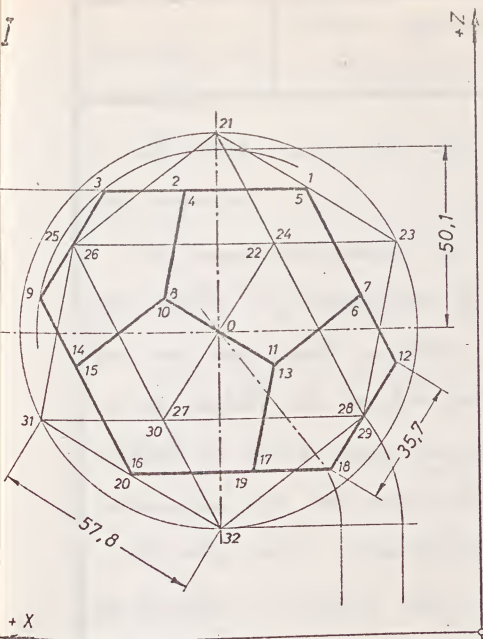
My dear Mr. [Name],

I have just received your letter of the 10th inst. and am glad to hear that you are well. I am also well and hope this letter finds you the same. I have been thinking of you very much lately and wondering how you are getting on. I hope you are still as active as ever. I have been very busy lately but I have managed to find some time to write you. I am sure you will be interested to hear from me. I have been thinking of you very much lately and wondering how you are getting on. I hope you are still as active as ever. I have been very busy lately but I have managed to find some time to write you. I am sure you will be interested to hear from me.

Yours truly,  
[Signature]







ENUNCIADO

Representar por el método gráti-co-analítico, en los planos I, II y III, el dodecaedro conjugado de un ico-saedro regular de 57,8 mm de lado, obtenido aquél al trazar por los puntos medios de las aristas del ico-saedro dado, rectas perpendiculares al plano determinado por cada una de sus aristas y el centro O del mis-mo, cuyas coordenadas son: O (72, 72, 85) mm.

Calcular previamente sus cotas fun-damentales que servirán de compro-bación al trazado gráfico.

Dibujar en formato A3v y a es-cala 1:1.

NUMERACIÓN DE VÉRTICES

Icosaedro dato..... 21 al 32  
Dodecaedro conjugado..... 1 al 20

	Propuesta	De entrega	Entregada	Califi-cación	(firma)	Escuela
Fecha:						Curso
Alumno:						
Escala	Poliedros regulares convexos conjugados					Lámina 17
1:1						Curso 19 -19



El poliedro conjugado de un icosaedro regular, es un dodecaedro regular.

El conjugado pedido con las condiciones del enunciado tiene sus aristas perpendiculares a las del icosaedro, y ambas se cortan en sus puntos medios. Por consiguiente, la esfera tangente a las aristas de ambos poliedros, es común, y el punto de contacto de dicha esfera con las aristas es precisamente el punto de intersección de las aristas.

Por otra parte, si proyectamos ortogonalmente los vértices del dodecaedro conjugado sobre las respectivas caras del icosaedro dado, dicha proyección coincide con el centro de cada cara triangular del icosaedro.

Estas propiedades nos permiten resolver fácilmente la representación propuesta en el enunciado.

### PROCESO GRÁFICO

Procedamos previamente a la representación del icosaedro dado en los planos I, II y III cuyo lado, conocido, es de 57,8 mm. Para ello utilizaremos el trazado dado en el "Proceso gráfico" de la lámina 5, determinando previamente el valor del radio  $\rho_{20}$  de la esfera circunscrita (ver Proceso gráfico-analítico) cuyo valor será:

$$\rho_{20} = 0,951057 \times 57,8 = 55,0 \text{ mm}$$

Seguidamente, completaremos en el plano II la proyección del dodecaedro pedido, cuyo contorno es coincidente con el del





icosaedro dado, y las aristas de las bases, perpendiculares en el punto medio a las del icosaedro (se conserva la perpendicularidad en el plano II por ser dichas aristas de las bases, paralelas a II). Con ello se consigue la representación completa en el plano II.

La propiedad, que se conserva en proyecciones, de bisecarse las aristas de ambos poliedros, y teniendo la representación del dodecaedro conjugado en II, permite realizar la del mismo en el plano I, y con ambas proyecciones la del plano III.

Con la numeración adecuada de los vértices y centros de ambos poliedros quedará completada la representación pedida.

#### PROCESO GRÁFICO-ANALÍTICO

El cálculo analítico de las magnitudes acotadas en la lámina 17 se desarrolla basándose en el efectuado en la lámina 4, en función del lado  $l'_{12}$  del icosaedro conjugado.

Primamente se determinará el radio  $a'_{12}$  de la esfera inscrita al mismo, teniendo en cuenta la igualdad en ambos poliedros, de la esfera tangente a las aristas.

El radio  $b_{20}$  de la esfera tangente a las aristas del icosaedro dado, tiene por valor, en función de  $l_{20}$

$$b_{20} = \frac{1 + \sqrt{5}}{4} l_{20} = b'_{12} \quad [1]$$

y por otra parte, el lado  $l'_{12}$  del dodecaedro conjugado, en fun-



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ción de  $l'_{12}$  es

$$l'_{12} = \frac{4}{3 + \sqrt{5}} \quad b'_{12} = \frac{4}{3 + \sqrt{5}} \times \frac{1 + \sqrt{5}}{4} l_{20} = \frac{1 + \sqrt{5}}{3 + \sqrt{5}} l_{20} = \frac{(1 + \sqrt{5})(3 - \sqrt{5})}{4} l_{20} =$$

$$= \frac{3 + 3\sqrt{5} - \sqrt{5} - 5}{4} l_{20} = \frac{2\sqrt{5} - 2}{4} l_{20} = \frac{\sqrt{5} - 1}{2} l_{20} \quad [2]$$

conocido  $l'_{12}$  en función de  $l_{20}$ , se calcularán los siguientes valores:  
(ver talla figura 4):

$$l'_{12} = \frac{\sqrt{5} - 1}{2} l_{20} = 0,618034 \times 57,830393 = 35,741147$$

siendo la relación  $\frac{l'_{12}}{l_{20}} = \frac{\sqrt{5} - 1}{2} = 0,618034 \dots$  [3]

$$y \quad a'_{12} = 1,401259 \times 35,741147 = 50,1 \text{ mm}$$

$$b'_{12} = 1,309017 \times \text{id.} = 46,8 \text{ "}$$

$$c'_{12} = 1,113516 \times \text{id.} = 39,8 \text{ "}$$

$$d'_{12} = 0,850651 \times \text{id.} = 30,4 \text{ "}$$

$$2\varphi_{12} = 116^\circ 33' 54,2''$$

$$e'_{12} = 1,376382 \times \text{id.} = 49,2 \text{ "}$$

$$f'_{12} = 0,525731 \times \text{id.} = 18,8 \text{ "}$$

$$g'_{12} = 0,850651 \times \text{id.} = 30,4 \text{ "}$$

\*

El valor más aproximado del lado  $l_{20}$  dado, se obtiene del exacto  $a_{20} = 55 \text{ mm.}$ ; de donde  $l_{20} = 55 : 0,951057 = 57,830393$  y de aquí:  $b_{20} = 57,830393 \times 0,809017 = 46,785769$ , por lo que será:  
así:  $l'_{12} = 46,785769 : 1,309017 = 35,741147$

1. The first part of the paper is devoted to a general discussion of the problem.

2. In the second part, we consider the case of a single particle.

3. The third part is devoted to the case of a system of particles.

4. In the fourth part, we consider the case of a continuous medium.

5. The fifth part is devoted to the case of a system of continuous media.

6. In the sixth part, we consider the case of a single continuous medium.

7. The seventh part is devoted to the case of a system of continuous media.

8. In the eighth part, we consider the case of a single continuous medium.

9. The ninth part is devoted to the case of a system of continuous media.

10. In the tenth part, we consider the case of a single continuous medium.

11. The eleventh part is devoted to the case of a system of continuous media.

12. In the twelfth part, we consider the case of a single continuous medium.

$$l'_{12} = 0,850651 \times 35,741147 = 30,4 \text{ mm}$$

$$l'_{20} = 0,688191 \times id = 24,6 \text{ "}$$

Verifícase que siendo

$$e'_{12} = \sqrt{\frac{5+2\sqrt{5}}{5}} l'_{12}$$

}

$$e_{20} = \sqrt{\frac{5+\sqrt{5}}{10}}$$

y según [3]  $\frac{l'_{12}}{l'_{20}} = \frac{\sqrt{5}-1}{2}$ , se verificará que

$$e'_{12} = \sqrt{\frac{5+2\sqrt{5}}{5}} l'_{12} = \sqrt{\frac{5+2\sqrt{5}}{5}} \times \frac{\sqrt{5}-1}{2} l'_{20} = \sqrt{\frac{5+\sqrt{5}}{10}} l'_{20}$$

de donde

$$e'_{12} = e_{20}$$

El desarrollo del cálculo anterior es el siguiente:

$$\begin{aligned} e'_{12} &= \sqrt{\frac{5+2\sqrt{5}}{5}} l'_{12} = \sqrt{\frac{5+2\sqrt{5}}{5}} \times \frac{\sqrt{5}-1}{2} l'_{20} = \sqrt{\frac{(5+2\sqrt{5})(\sqrt{5}-1)^2}{20}} l'_{20} = \\ &= \sqrt{\frac{(5+2\sqrt{5})(5+1-2\sqrt{5})}{20}} l'_{20} = \sqrt{\frac{(5+2\sqrt{5})(3-\sqrt{5})}{10}} l'_{20} = \sqrt{\frac{15+6\sqrt{5}-5\sqrt{5}-10}{10}} l'_{20} = \\ &= \sqrt{\frac{5+\sqrt{5}}{10}} l'_{20} \end{aligned}$$

lo cual nos demuestra que: "El radio de la circunferencia circunscrita al decágono regular, contorno en II del dodecaedro conjugado, es igual al del icosaedro dado en la misma proyección".

De aquí se deduce que los lados de ambos decágonos

Let  $x = \cos \theta$  then  $dx = -\sin \theta d\theta$

Now we have

$$\frac{1}{\sqrt{1-x^2}} = \frac{1}{\sqrt{1-\cos^2 \theta}} = \frac{1}{\sin \theta}$$

$$\int \frac{1}{\sqrt{1-x^2}} = \int \frac{1}{\sin \theta} \cdot (-\sin \theta d\theta) = -\int \frac{1}{\sin \theta} d\theta$$

$$= -\int \csc \theta d\theta = -\ln |\csc \theta - \cot \theta| + C$$

$$= -\ln \left| \frac{1}{\sin \theta} - \frac{\cos \theta}{\sin \theta} \right| + C$$

Now we have  $\sin \theta = \sqrt{1-x^2}$  and  $\cos \theta = x$

$$= -\ln \left| \frac{1}{\sqrt{1-x^2}} - \frac{x}{\sqrt{1-x^2}} \right| + C$$

$$= -\ln \left| \frac{1-x}{\sqrt{1-x^2}} \right| + C = -\ln \left| \frac{1-x}{\sqrt{(1-x)(1+x)}} \right| + C$$

$$= \ln \left| \frac{\sqrt{1+x}}{1-x} \right| + C$$

Therefore, the integral of  $\frac{1}{\sqrt{1-x^2}}$  is  $\ln \left| \frac{\sqrt{1+x}}{1-x} \right| + C$ .

Q.E.D.



son también iguales, por lo que

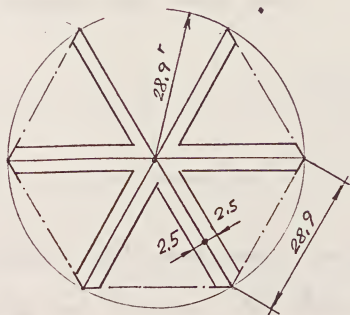
$$l'_{12} = l'_{20}$$

### FIGURA CORPÓREA

El dodecaedro conjugado se obtiene por el acoplamiento de 12 pentágonos regulares de 35.7 mm de lado.

El icosaedro dado se obtiene formando previamente 12 pirámides pentagonales (transparentes y sin base), cuyas caras laterales son 5 triángulos equiláteros de 28.9 mm (mitad del lado del icosaedro). Estas pirámides se acoplarán sobre cada una de las doce caras del conjugado, de forma que sus vértices estén en los puntos medios de las aristas de aquél.

También puede obtenerse el icosaedro dado, sustituyendo las caras transparentes de las 12 pirámides pentagonales, por las obtenidas según el trazado que se representa a continuación:



Estas 12 pirámides se acoplarán al dodecaedro conjugado, colocando sus vértices A, B, C, D, E, en los puntos medios de las aristas

...

...

...

...

...



...

RESUMEN DEL CÁLCULO DE MAGNITUDES DE ESTE EJERCICIONomenclatura empleada: $l'_{12}$  = Lado del dodecaedro conjugado. $a'_{12}$  = Radio de la esfera circunscrita al mismo. $b'_{12}$  = Radio de la esfera tangente a las aristas. $c'_{12}$  = Radio de la esfera inscrita. $d'_{12}$  = Radio de la circunferencia circunscrita al polígono de una cara. $2\varphi'_{12}$  = Ángulo rectilíneo del diedro formado por dos caras contiguas. $e'_{12}$  = Radio de la circunferencia circunscrita al decágono regular, contorno de la vista II. $f'_{12}$  = Altura intermedia del contorno de las vistas I y III. $g'_{12}$  = Alturas extremas del contorno de las vistas I y III. $l'_{12}$  = Lado del decágono regular, contorno de la vista II. $k'_{12}$  = Apotema del polígono de una cara. $S'_{12}$  = Superficie. $V'_{12}$  = Volumen.

Los valores anteriores del dodecaedro conjugado, pueden obtenerse directamente en función del lado  $l'_{20}$  del icosaedro dado, ya que el valor conocido de  $l'_{12}$  en función de  $l'_{20}$  (ver fórmula [2]), es

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$$l'_{12} = \frac{\sqrt{5}-1}{2} l_{20}$$

Sustituyendo este valor en las fórmulas 30 a 41 de la lámina 4, tendremos:

$$a'_{12} = \frac{\sqrt{15} + \sqrt{3}}{4} \times \frac{\sqrt{5}-1}{2} l_{20} = \frac{\sqrt{3}}{2} l_{20}$$

$$\begin{aligned} \text{Desarrollo del cálculo anterior: } a'_{12} &= \frac{\sqrt{15} + \sqrt{3}}{4} \times \frac{\sqrt{5}-1}{2} l_{20} = \\ &= \frac{\sqrt{75} + \sqrt{15} - \sqrt{15} - \sqrt{3}}{8} l_{20} = \frac{5\sqrt{3} - \sqrt{3}}{8} l_{20} = \boxed{\frac{\sqrt{3}}{2} l_{20}} \end{aligned}$$

$$b'_{12} = \frac{3 + \sqrt{5}}{4} \times \frac{\sqrt{5}-1}{2} l_{20} = \frac{1 + \sqrt{5}}{4} l_{20}$$

$$\begin{aligned} \text{Desarrollo del cálculo anterior: } b'_{12} &= \frac{3 + \sqrt{5}}{4} \times \frac{\sqrt{5}-1}{2} l_{20} = \\ &= \frac{3\sqrt{5} + 5 - 3 - \sqrt{5}}{8} l_{20} = \frac{2\sqrt{5} + 2}{8} l_{20} = \boxed{\frac{1 + \sqrt{5}}{4} l_{20}} \end{aligned}$$

$$c'_{12} = \sqrt{\frac{11\sqrt{5} + 25}{40}} \times \frac{\sqrt{5}-1}{2} l_{20} = \sqrt{\frac{5 + 2\sqrt{5}}{20}} l_{20}$$

$$\begin{aligned} \text{Desarrollo del cálculo anterior: } c'_{12} &= \sqrt{\frac{11\sqrt{5} + 25}{40}} \times \frac{\sqrt{5}-1}{2} l_{20} = \\ &= \sqrt{\frac{(11\sqrt{5} + 25)(\sqrt{5}-1)^2}{40 \times 4}} l_{20} = \sqrt{\frac{(11\sqrt{5} + 25)(5 + 1 - 2\sqrt{5})}{40 \times 4}} l_{20} = \end{aligned}$$

$$= \sqrt{\frac{(11\sqrt{5} + 25)(3 - \sqrt{5})}{40 \times 2}} l_{20} = \sqrt{\frac{33\sqrt{5} + 75 - 55 - 25\sqrt{5}}{40 \times 2}} l_{20} = \sqrt{\frac{20 + 8\sqrt{5}}{80}} l_{20} =$$

$$= \boxed{\sqrt{\frac{5 + 2\sqrt{5}}{20}} l_{20}}$$



$$x^2 + y^2 = z^2$$

Let  $x = a^2 - b^2$ ,  $y = 2ab$ ,  $z = a^2 + b^2$

$$x^2 + y^2 = (a^2 - b^2)^2 + (2ab)^2$$

$= a^4 - 2a^2b^2 + b^4 + 4a^2b^2$

$$= a^4 + 2a^2b^2 + b^4$$

$$= (a^2 + b^2)^2$$

$\therefore x^2 + y^2 = z^2$

$$\therefore (a^2 - b^2)^2 + (2ab)^2 = (a^2 + b^2)^2$$

$$\therefore x^2 + y^2 = z^2$$

Let  $x = a^2 - b^2$ ,  $y = 2ab$ ,  $z = a^2 + b^2$

$$x^2 + y^2 = (a^2 - b^2)^2 + (2ab)^2$$

$$= a^4 - 2a^2b^2 + b^4 + 4a^2b^2$$

$$= (a^2 + b^2)^2$$

$$d'_{12} = \sqrt{\frac{5+\sqrt{5}}{10}} \times \frac{\sqrt{5}-1}{2} l_{20} = \sqrt{\frac{5-\sqrt{5}}{10}} l_{20}$$

$$\begin{aligned} \text{Desarrollo del cálculo anterior: } d'_{12} &= \sqrt{\frac{5+\sqrt{5}}{10}} \times \frac{\sqrt{5}-1}{2} l_{20} = \\ &= \sqrt{\frac{(5+\sqrt{5})(\sqrt{5}-1)^2}{10 \times 4}} l_{20} = \sqrt{\frac{(5+\sqrt{5})(5+1-2\sqrt{5})}{40}} l_{20} = \sqrt{\frac{(5+\sqrt{5})(3-\sqrt{5})}{20}} l_{20} = \\ &= \sqrt{\frac{15+3\sqrt{5}-5\sqrt{5}-5}{20}} l_{20} = \sqrt{\frac{10-2\sqrt{5}}{20}} l_{20} = \boxed{\sqrt{\frac{5-\sqrt{5}}{10}} l_{20}} \end{aligned}$$

$$e'_{12} = \sqrt{\frac{5+2\sqrt{5}}{5}} \times \frac{\sqrt{5}-1}{2} l_{20} = \sqrt{\frac{5+\sqrt{5}}{10}} l_{20}$$

$$\begin{aligned} \text{Desarrollo del cálculo anterior: } e'_{12} &= \sqrt{\frac{5+2\sqrt{5}}{5}} \times \frac{\sqrt{5}-1}{2} l_{20} = \\ &= \sqrt{\frac{(5+2\sqrt{5})(\sqrt{5}-1)^2}{5 \times 4}} l_{20} = \sqrt{\frac{(5+2\sqrt{5})(5+1-2\sqrt{5})}{20}} l_{20} = \sqrt{\frac{(5+2\sqrt{5})(3-\sqrt{5})}{10}} l_{20} = \\ &= \sqrt{\frac{15+6\sqrt{5}-5\sqrt{5}-10}{10}} l_{20} = \boxed{\sqrt{\frac{5+\sqrt{5}}{10}} l_{20}} \end{aligned}$$

$$f'_{12} = \sqrt{\frac{5-\sqrt{5}}{10}} \times \frac{\sqrt{5}-1}{2} l_{20} = \sqrt{\frac{5-2\sqrt{5}}{5}} l_{20}$$

$$\begin{aligned} \text{Desarrollo del cálculo anterior: } f'_{12} &= \sqrt{\frac{5-\sqrt{5}}{10}} \times \frac{\sqrt{5}-1}{2} l_{20} = \\ &= \sqrt{\frac{(5-\sqrt{5})(\sqrt{5}-1)^2}{10 \times 4}} l_{20} = \sqrt{\frac{(5-\sqrt{5})(5+1-2\sqrt{5})}{40}} l_{20} = \sqrt{\frac{(5-\sqrt{5})(3-\sqrt{5})}{20}} l_{20} = \\ &= \sqrt{\frac{15-3\sqrt{5}-5\sqrt{5}+5}{20}} l_{20} = \sqrt{\frac{20-8\sqrt{5}}{20}} l_{20} = \boxed{\sqrt{\frac{5-2\sqrt{5}}{5}} l_{20}} \end{aligned}$$

$$g'_{12} = i'_{12} = d'_{12} = \sqrt{\frac{5-\sqrt{5}}{10}} l_{20}$$



$$k'_{12} = \sqrt{\frac{5+2\sqrt{5}}{20}} \times \frac{\sqrt{5}-1}{2} l_{20} = \frac{1}{2} \sqrt{\frac{5+\sqrt{5}}{10}} l_{20} = \frac{1}{2} e'_{12}$$

$$\begin{aligned} \text{Desarrollo del cálculo anterior: } k'_{12} &= \sqrt{\frac{5+2\sqrt{5}}{20}} \times \frac{\sqrt{5}-1}{2} l_{20} = \\ &= \sqrt{\frac{(5+2\sqrt{5})(\sqrt{5}-1)^2}{20 \times 4}} l_{20} = \sqrt{\frac{(5+2\sqrt{5})(5+1-2\sqrt{5})}{80}} l_{20} = \sqrt{\frac{(5+2\sqrt{5})(3-\sqrt{5})}{40}} l_{20} = \\ &= \sqrt{\frac{15+6\sqrt{5}-5\sqrt{5}-10}{40}} l_{20} = \sqrt{\frac{5+\sqrt{5}}{40}} l_{20} = \boxed{\frac{1}{2} \sqrt{\frac{5+\sqrt{5}}{10}} l_{20}} \end{aligned}$$

$$S'_{12} = 3 \sqrt{25+10\sqrt{5}} \times \left( \frac{\sqrt{5}-1}{2} l_{20} \right)^2 = \frac{3}{2} \sqrt{10(5-\sqrt{5})} l_{20}^2$$

$$\begin{aligned} \text{Desarrollo del cálculo anterior: } S'_{12} &= 3 \sqrt{25+10\sqrt{5}} \times \left( \frac{\sqrt{5}-1}{2} l_{20} \right)^2 = \\ &= 3 \sqrt{25+10\sqrt{5}} \times \frac{(\sqrt{5}-1)^2}{4} l_{20}^2 = 3 \sqrt{25+10\sqrt{5}} \times \frac{6-2\sqrt{5}}{4} l_{20}^2 = \\ &= \frac{3}{2} \sqrt{(25+10\sqrt{5})(3-\sqrt{5})^2} l_{20}^2 = \frac{3}{2} \sqrt{(25+10\sqrt{5})(9+5-6\sqrt{5})} l_{20}^2 = \\ &= \frac{3}{2} \sqrt{(25+10\sqrt{5}) \times 2 \times (7-3\sqrt{5})} l_{20}^2 = \frac{3}{2} \sqrt{2 \times (175+70\sqrt{5}-75\sqrt{5}-150)} l_{20}^2 = \\ &= \frac{3}{2} \sqrt{2 \times (25-5\sqrt{5})} l_{20}^2 = \boxed{\frac{3}{2} \sqrt{10 \times (5-\sqrt{5})} l_{20}^2} \end{aligned}$$

$$V'_{12} = \frac{7\sqrt{5}+15}{4} \times \left( \frac{\sqrt{5}-1}{2} l_{20} \right)^3 = \frac{5+\sqrt{5}}{4} l_{20}^3$$

$$\begin{aligned} \text{Desarrollo del cálculo anterior: } V'_{12} &= \frac{7\sqrt{5}+15}{4} \times \left( \frac{\sqrt{5}-1}{2} l_{20} \right)^3 = \\ &= \frac{7\sqrt{5}+15}{4} \times \frac{(\sqrt{5}-1)^3}{8} l_{20}^3 = \frac{7\sqrt{5}+15}{4} \times \frac{5\sqrt{5}-15+3\sqrt{5}-1}{8} l_{20}^3 = \end{aligned}$$





$$= \frac{7\sqrt{5} + 15}{4} \cdot \frac{8\sqrt{5} - 16}{8} l_{20}^3 = \frac{(7\sqrt{5} + 15)(\sqrt{5} - 2)}{4} l_{20}^3 = \frac{35 + 15\sqrt{5} - 14\sqrt{5} - 30}{4} l_{20}^3 =$$

$$= \frac{5 + \sqrt{5}}{4} l_{20}^3$$

Son útiles las siguientes relaciones:

a)  $\frac{l_{20}}{l'_{12}} = \frac{2}{\sqrt{5} - 1} = \frac{2(\sqrt{5} + 1)}{4} = \frac{\sqrt{5} + 1}{2}$  (ver fórmula [2])

b)  $\frac{S_{20}}{S'_{12}} = \frac{5\sqrt{3}}{\frac{3}{2}\sqrt{10}(\sqrt{5} - \sqrt{5})} \frac{l_{20}^2}{l_{20}^2} = \sqrt{\frac{5 + \sqrt{5}}{6}}$



Desarrollo del cálculo anterior:  $\frac{S_{20}}{S'_{12}} = \frac{5\sqrt{3}}{\frac{3}{2}\sqrt{10}(\sqrt{5} - \sqrt{5})} \frac{l_{20}^2}{l_{20}^2} =$   
 $= \frac{10\sqrt{3}}{3\sqrt{10}(\sqrt{5} - \sqrt{5})} = \frac{10\sqrt{3}\sqrt{10}(\sqrt{5} - \sqrt{5})}{3 \times 10 \times (\sqrt{5} - \sqrt{5})} = \frac{\sqrt{30}(\sqrt{5} - \sqrt{5})}{3(\sqrt{5} - \sqrt{5})} = \frac{\sqrt{30}(\sqrt{5} - \sqrt{5}) \cdot (\sqrt{5} + \sqrt{5})}{3 \times 20} =$   
 $= \frac{\sqrt{30}(\sqrt{5} - \sqrt{5})(\sqrt{5} + \sqrt{5})^2}{60} = \frac{\sqrt{30 \times 20 \times (\sqrt{5} + \sqrt{5})^2}}{60} = \frac{\sqrt{6 \times (\sqrt{5} + \sqrt{5})^2}}{6} = \boxed{\sqrt{\frac{5 + \sqrt{5}}{6}}}$

c)  $\frac{V_{20}}{V'_{12}} = \frac{\frac{15 + 5\sqrt{5}}{12} l_{20}^3}{\frac{5 + \sqrt{5}}{4} l_{20}^3} = \frac{5 + \sqrt{5}}{6}$

Desarrollo del cálculo anterior:  $\frac{V_{20}}{V'_{12}} = \frac{\frac{15 + 5\sqrt{5}}{12} l_{20}^3}{\frac{5 + \sqrt{5}}{4} l_{20}^3} =$   
 $= \frac{4(15 + 5\sqrt{5})}{12(5 + \sqrt{5})} = \frac{(15 + 5\sqrt{5})(\sqrt{5} - \sqrt{5})}{3 \times 20} = \frac{75 + 25\sqrt{5} - 15\sqrt{5} - 25}{60} = \frac{50 + 10\sqrt{5}}{60} =$   
 $= \boxed{\frac{5 + \sqrt{5}}{6}}$

25

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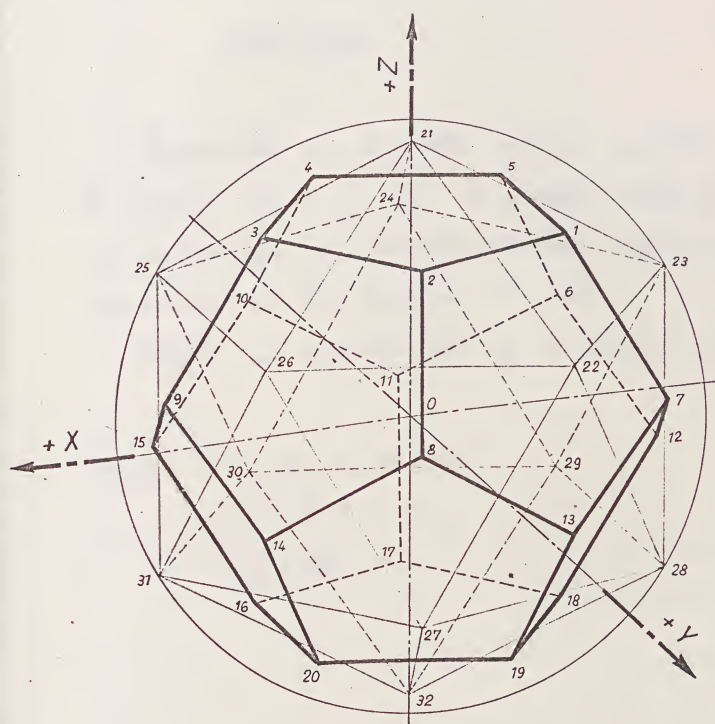
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En el cuadro sinóptico dado a continuación, resumimos los resultados anteriores.

CUADRO SINÓPTICO

Magnitud	Valor exacto	Valor decimal aproximado
(181) $l'_{12}$	$\frac{\sqrt{5}-1}{2} l_{20}$	0, 61 80 34 ---- $l_{20}$
(182) $a'_{12}$	$\frac{\sqrt{3}}{2} l_{20}$	0, 86 60 25 ---- $l_{20}$
(183) $b'_{12}$	$\frac{1+\sqrt{5}}{4} l_{20}$	0, 80 90 17 ---- $l_{20}$
(184) $c'_{12}$	$\sqrt{\frac{5+2\sqrt{5}}{20}} l_{20}$	0, 68 81 91 ---- $l_{20}$
(185) $d'_{12}$	$\sqrt{\frac{5-\sqrt{5}}{10}} l_{20}$	0, 52 57 31 ---- $l_{20}$
(186) $2 \varphi'_{12}$	$\text{sen } \varphi'_{12} = \sqrt{\frac{5+\sqrt{5}}{10}}$	$\text{sen } \varphi_{12} = 0, 85 06 51$ $2 \varphi_{12} = 116^{\circ} 33' 54,2''$
(187) $e'_{12}$	$\sqrt{\frac{5+\sqrt{5}}{10}} l_{20}$	0, 85 06 51 --- $l_{20}$
(188) $f'_{12}$	$\sqrt{\frac{5-2\sqrt{5}}{5}} l_{20}$	0, 32 49 20 ... $l_{20}$
(189) $g'_{12}$	$\sqrt{\frac{5-\sqrt{5}}{10}} l_{20}$	0, 52 57 31 ---- $l_{20}$
(190) $l'_{12}$	$\sqrt{\frac{5-\sqrt{5}}{20}} l_{20}$	0, 52 57 31 ---- $l_{20}$
(191) $k'_{12}$	$\frac{1}{2} \sqrt{\frac{5+\sqrt{5}}{10}} l_{20}$	0, 42 53 25 --- $l_{20}$
(192) $s'_{12}$	$\frac{3}{2} \sqrt{10 \times (5-\sqrt{5})} l_{20}^2$	7, 88 59 67 --- $l_{20}^2$
(193) $v'_{12}$	$\frac{5+\sqrt{5}}{4} l_{20}^3$	1, 80 90 17 ---- $l_{20}^3$
Relaciones entre magnitudes		
$l_{20} : l'_{12}$	(194) $\frac{\sqrt{5}+1}{2}$	1, 61 80 34 ...
$S_{20} : S'_{12}$	(195) $\sqrt{\frac{5+\sqrt{5}}{6}}$	1, 09 81 85 ...
$V_{20} : V'_{12}$	(196) $\frac{5+\sqrt{5}}{6}$	1, 20 60 11 ----





*Poliedros regulares convexos conjugados*





Lámina 18

## ENUNCIADO

Representar por el método gráfico-analítico, en los planos I, II, y III, el poliedro obtenido por la intersección de los dos poliedros conjugados representados en las láminas 16 y 17.

Dibujar en formato A3V y a escala 1:1.

DATOS

$$O (72, 72, 85) \text{ mm}$$

$$l_{12} = 35,7 \text{ mm}$$

$$l'_{20} = 57,8 \text{ mm}$$

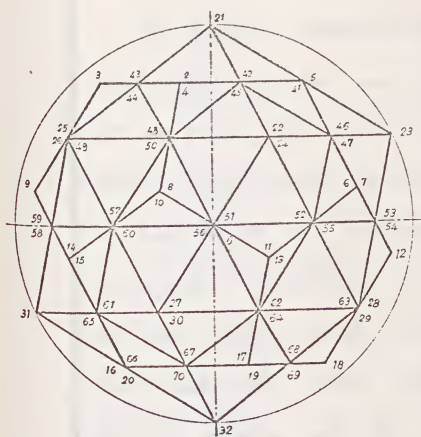
Editor:

Dear Sir:

I have the honor to acknowledge the receipt of your letter of the 28th inst. in relation to the matter of the American Medical Association's position on the subject of the registration of medical practitioners. I am sorry that I cannot give you a more definite answer at this time, but I am sure that the Association's position is well known to you.

Very respectfully,  
 J. H. H. H.  
 Secretary

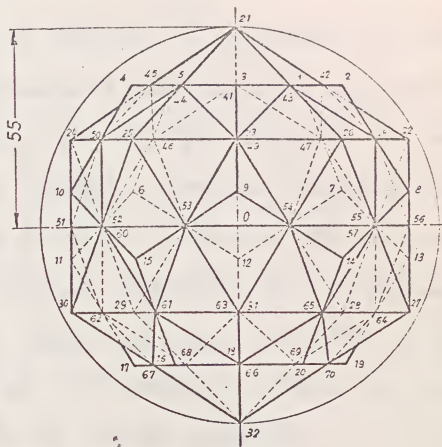
I



+X

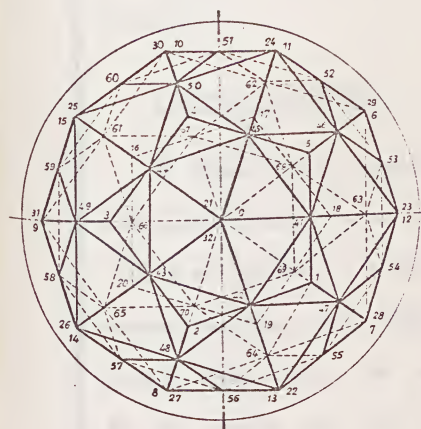
+Z

III



O

+Y



+Y

### NUMERACIÓN DE VÉRTICES

Dodecaedro regular ..... 1 al 20

kosaedro regular ..... 21 al 32

Núcleo (arquimediano IV)... 41 al 70

### ENUNCIADO

Representar por el método gráfico-analítico, en los planos I, II y III, el poliedro obtenido por la intersección de los dos poliedros conjugados estudiados en las láminas 16 y 17.

Dibujar en formato A3v y a escala 1:1.

Propuesta	De entrega	Entregada	Calificación	(firma)	Escuela
Fecha:					Curso
Alumno:					
Escala	Poliedros regulares convexos conjugados				Lámina 18
1:1					Curso 19 -19

II





El poliedro representado en la lámina 18, intersección de un dodecaedro regular con un icosaedro también regular, conjugado del anterior, de centro común, y con la esfera tangente a las aristas igualmente común a ambos, se obtiene completando el trazado de la lámina 16 (o el de la lámina 17), con los triángulos equiláteros y pentágonos regulares que se forman al ser cortado cada ángulo sólido de uno de ellos por las caras del otro. Los vértices de estos polígonos estarán en los puntos medios de las aristas de ambos poliedros.

El "Proceso gráfico" y el "Proceso gráfico-analítico" de la mencionada lámina 16. (o de la 17), sirve igualmente para este ejercicio.

#### FIGURA CORPÓREA

Se obtiene por acoplamiento de 12 pirámides rectas (sin base), compuesta cada una de ellas por 5 triángulos equiláteros de 28,9 mm de lado (mitad de  $l_{20} = 57,8$  mm) correspondientes al icosaedro regular, con otras 20 pirámides rectas (sin base), compuesta cada una de ellas por 3 triángulos rectángulos isósceles de catetos de 17,9 mm de lado (mitad de  $l'_{12} = 35,7$ ) e hipotenusa de 28,9 mm; correspondientes estas últimas al dodecaedro conjugado.

Este poliedro es cóncavo ya que la prolongación del plano de cualquier cara de uno de los poliedros, deja en distintos semiespacios a los vértices del otro.



CALCULO DE MAGNITUDES DE ESTE POLIEDRO

El núcleo de este poliedro es el sólido común en la intersección de un dodecaedro regular con su icosaedro conjugado con respecto a los puntos medios de sus aristas.

Este sólido común es un poliedro no regular, convexo, compuesto de 12 pentágonos regulares y 20 triángulos equiláteros de lado de igual longitud en ambos polígonos. Los ángulos poliedros (30 en total) son todos iguales y en cada uno de ellos concurren dos caras pentagonales y dos triangulares en forma alternada. Este poliedro entra en el grupo de los llamados "Arquimedianos", cuyo estudio y representación desarrollamos en las láminas 33 a 48. Corresponde al designado por "Arquimediano IV" (lámina 36), donde se detalla el cálculo analítico de sus principales magnitudes, algunas de ellas de aplicación en el poliedro estudiado en esta lámina.

$\alpha$  = Ángulo rectilíneo del diedro formado por dos caras secantes contiguas.

El diedro  $\alpha$ , formado por una cara del dodecaedro con la cara secante contigua del icosaedro conjugado es opuesto por el vértice al diedro  $\varphi_{3.5}$  del Arquimediano IV que forma su núcleo, y por consiguiente de igual magnitud que éste; su valor será pues: (ver lámina )

It is a well-known fact that the American people are not properly educated in the principles of medicine. The average man and woman know very little of the science of medicine, and are consequently easily misled by quacks and charlatans. It is the duty of the medical profession to educate the public in the principles of medicine, and to show them the value of the services of the physician.

The first step in the education of the public is to show them the value of the services of the physician. The physician is not a mere tradesman, who takes money for his services. He is a man of science, who has spent years of study and labor to acquire his knowledge. He is a man of honor, who is bound by a sacred oath to do no harm to his patients. He is a man of skill, who has learned the art of healing. The physician is a man of great importance to the community, and his services should be valued accordingly.

The second step in the education of the public is to show them the value of the services of the nurse. The nurse is not a mere attendant, who takes care of the patient's needs. She is a woman of science, who has spent years of study and labor to acquire her knowledge. She is a woman of honor, who is bound by a sacred oath to do no harm to her patients. She is a woman of skill, who has learned the art of nursing. The nurse is a woman of great importance to the community, and her services should be valued accordingly.

The third step in the education of the public is to show them the value of the services of the pharmacist. The pharmacist is not a mere tradesman, who takes money for his services. He is a man of science, who has spent years of study and labor to acquire his knowledge. He is a man of honor, who is bound by a sacred oath to do no harm to his patients. He is a man of skill, who has learned the art of compounding. The pharmacist is a man of great importance to the community, and his services should be valued accordingly.



$\alpha =$ 

[1]

S = Área lateral en función de  $l_{20}$

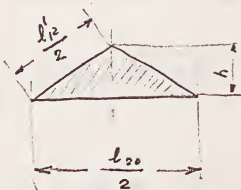
La superficie lateral está compuesta:

- 1) De 12 pirámides  $P_5$ , cuyas caras laterales son 5 triángulos equiláteros de lado igual a  $\frac{l_{20}}{2}$ , y
- 2) De 20 pirámides  $P_3$ , cuyas caras laterales son 3 triángulos isósceles de base igual a  $\frac{l_{20}}{2}$  y lados contiguos a la base (iguales entre sí), iguales a  $\frac{l'_{12}}{2}$

El área lateral de las 12 pirámides  $P_5$ , será pues

$$S_1 = 12 \times 5 \times \frac{\sqrt{3}}{4} \left( \frac{l_{20}}{2} \right)^2 = \frac{15\sqrt{3}}{4} l_{20}^2 \quad (\text{compuestos})$$

Para el cálculo del área lateral de  $P_3$ , determinemos previamente la altura de cada una de sus caras (ver figura), en la que (ver lám. 17, fórmula 181)



$$\frac{l'_{12}}{2} = \frac{\frac{\sqrt{5}-1}{2} l_{20}}{2} = \frac{\sqrt{5}-1}{4} l_{20}$$

de donde

$$h = \sqrt{\left( \frac{l'_{12}}{2} \right)^2 - \left( \frac{l_{20}}{4} \right)^2} = \sqrt{\left( \frac{\sqrt{5}-1}{4} l_{20} \right)^2 - \frac{1}{16} l_{20}^2} = \frac{\sqrt{5-2\sqrt{5}}}{4} l_{20}$$

El desarrollo de este cálculo es el siguiente:

$$h = \sqrt{\left( \frac{\sqrt{5}-1}{4} l_{20} \right)^2 - \frac{1}{16} l_{20}^2} = \sqrt{\frac{5+1-2\sqrt{5}}{16} - \frac{1}{16}} \times l_{20} = \sqrt{\frac{6-2\sqrt{5}}{16} - \frac{1}{16}} \times l_{20}$$





$$= \sqrt{\frac{6-2\sqrt{5}-1}{16}} l_{20} = \boxed{\frac{\sqrt{5-2\sqrt{5}}}{4}} l_{20}$$

El área lateral de las 20 pirámides  $P_3$ , será pues

$$S_2 = 20 \times 3 \times \frac{l_{20}}{2} \times \frac{1}{2} \left( \frac{\sqrt{5-2\sqrt{5}}}{4} \right) l_{20} = \frac{15\sqrt{5-2\sqrt{5}}}{4} l_{20}^2$$

y el área total  $S$ ,

$$S = S_1 + S_2 = \frac{15\sqrt{3}}{4} l_{20}^2 + \frac{15\sqrt{5-2\sqrt{5}}}{4} l_{20}^2 = \frac{15\sqrt{3} + 15\sqrt{5-2\sqrt{5}}}{4} l_{20}^2 =$$

$$= \frac{15(\sqrt{3} + \sqrt{5-2\sqrt{5}})}{4} l_{20}^2 = 9,919725... l_{20}^2 \quad [2]$$

$S =$  Área lateral en función de  $l_2$

De la relación  $\frac{l_{20}}{l_{12}} = \frac{\sqrt{5}+1}{2}$  (ver lám. 17, fórm. 194)

se deduce que  $l_{20} = \frac{\sqrt{5}+1}{2} l_{12}$ , que sustituido en [2] nos da

$$S = \frac{15(\sqrt{3} + \sqrt{5-2\sqrt{5}})}{4} \times \left( \frac{\sqrt{5}+1}{2} l_{12} \right)^2 = \frac{15(3\sqrt{3} + \sqrt{15} + \sqrt{10+2\sqrt{5}})}{8} l_{12}^2 =$$

$$= 24,137553... l_{12}^2 \quad [3]$$

Desarrollo del cálculo anterior:

$$S = \frac{15(\sqrt{3} + \sqrt{5-2\sqrt{5}})}{4} \times \left( \frac{\sqrt{5}+1}{2} l_{12} \right)^2 = \frac{15}{4} (\sqrt{3} + \sqrt{5-2\sqrt{5}}) \times \frac{5+1+2\sqrt{5}}{4} l_{12}^2 =$$

Q. 1. Find the value of  $\sin^{-1}(\sin \frac{5\pi}{6})$

Sol. We know that  $\sin^{-1}(\sin x) = x$  if  $x$  lies in  $[-\frac{\pi}{2}, \frac{\pi}{2}]$

$$\therefore \sin^{-1}(\sin \frac{5\pi}{6}) = \frac{5\pi}{6}$$

Q. 2. Find the value of  $\cos^{-1}(\cos \frac{7\pi}{6})$

$$\cos^{-1}(\cos \frac{7\pi}{6}) = \frac{7\pi}{6}$$

$$\text{Q. 3. Find the value of } \tan^{-1}(\tan \frac{3\pi}{4})$$

$$\tan^{-1}(\tan \frac{3\pi}{4}) = \frac{3\pi}{4}$$

$$\text{Q. 4. Find the value of } \cot^{-1}(\cot \frac{5\pi}{6})$$

$$\cot^{-1}(\cot \frac{5\pi}{6}) = \frac{5\pi}{6}$$

$$\text{Q. 5. Find the value of } \sec^{-1}(\sec \frac{2\pi}{3})$$

$$\sec^{-1}(\sec \frac{2\pi}{3}) = \frac{2\pi}{3}$$

Q. 6. Find the value of  $\csc^{-1}(\csc \frac{4\pi}{3})$

$$\csc^{-1}(\csc \frac{4\pi}{3}) = \frac{4\pi}{3}$$

$$\begin{aligned}
 &= \frac{15}{8} (\sqrt{3} + \sqrt{5-2\sqrt{5}})(3 + \sqrt{5}) l_{12}^2 = \frac{15}{8} (3\sqrt{3} + \sqrt{15} + \sqrt{(5-2\sqrt{5})(3+\sqrt{5})^2}) l_{12}^2 = \\
 &= \frac{15}{8} (3\sqrt{3} + \sqrt{15} + \sqrt{(5-2\sqrt{5})(9+5+6\sqrt{5})}) l_{12}^2 = \frac{15}{8} (3\sqrt{3} + \sqrt{15} + \\
 &+ \sqrt{(5-2\sqrt{5})(7+3\sqrt{5}) \cdot 2}) l_{12}^2 = \frac{15}{8} (3\sqrt{3} + \sqrt{15} + \sqrt{2 \cdot (35-14\sqrt{5}+15\sqrt{5}-30)}) l_{12}^2 = \\
 &= \frac{15}{8} (3\sqrt{3} + \sqrt{15} + \sqrt{2(5+\sqrt{5})}) l_{12}^2 = \boxed{\frac{15(3\sqrt{3} + \sqrt{15} + \sqrt{10+2\sqrt{5}})}{8} l_{12}^2}
 \end{aligned}$$

$V$ : Volumen en función de  $l_{20}$

El volumen total de este poliedro se puede obtener como suma de volúmenes de uno de los poliedros regulares y de las pirámides complementarias formadas en cada una de las caras del primero por la intersección de los ángulos sólidos del otro.

1) Considerando como primer sumando el volumen del dodecaedro, éste será (ver lám. 4, fórm. 41):

$$V_{12} = \frac{7\sqrt{5} + 15}{4} l_{12}^3 \quad \text{y en función de } l_{20} \text{ (ver lám. 17,$$

fórm. 181)

$$V_{12} = \frac{7\sqrt{5} + 15}{4} \times \left( \frac{\sqrt{5}-1}{2} l_{20} \right)^3 = \frac{5+5\sqrt{5}}{4} l_{20}^3$$

$$\text{Desarrollo del cálculo anterior: } V_{12} = \frac{7\sqrt{5} + 15}{4} \times \left( \frac{\sqrt{5}-1}{2} l_{20} \right)^3 =$$

$$= \frac{7\sqrt{5} + 15}{4} \times \frac{5\sqrt{5} - 15 + 3\sqrt{5} - 1}{8} l_{20}^3 = \frac{7\sqrt{5} + 15}{4} \times \frac{8\sqrt{5} - 16}{8} l_{20}^3 =$$

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$$\begin{aligned}
 & \frac{1}{2} \left( \frac{1}{\sqrt{1-x^2}} + \frac{1}{\sqrt{1-x^2}} \right) = \frac{1}{\sqrt{1-x^2}} \\
 & \frac{1}{2} \left( \frac{1}{\sqrt{1-x^2}} + \frac{1}{\sqrt{1-x^2}} \right) = \frac{1}{\sqrt{1-x^2}} \\
 & \frac{1}{2} \left( \frac{1}{\sqrt{1-x^2}} + \frac{1}{\sqrt{1-x^2}} \right) = \frac{1}{\sqrt{1-x^2}}
 \end{aligned}$$

The first part of the proof is to show that the function  $f(x) = \frac{1}{\sqrt{1-x^2}}$  is an even function. This is done by showing that  $f(-x) = f(x)$ . The second part of the proof is to show that the function  $f(x) = \frac{1}{\sqrt{1-x^2}}$  is a solution to the differential equation  $y' = xy$ . This is done by substituting  $y = \frac{1}{\sqrt{1-x^2}}$  into the equation and simplifying.

$$\begin{aligned}
 & \frac{1}{2} \left( \frac{1}{\sqrt{1-x^2}} + \frac{1}{\sqrt{1-x^2}} \right) = \frac{1}{\sqrt{1-x^2}} \\
 & \frac{1}{2} \left( \frac{1}{\sqrt{1-x^2}} + \frac{1}{\sqrt{1-x^2}} \right) = \frac{1}{\sqrt{1-x^2}} \\
 & \frac{1}{2} \left( \frac{1}{\sqrt{1-x^2}} + \frac{1}{\sqrt{1-x^2}} \right) = \frac{1}{\sqrt{1-x^2}}
 \end{aligned}$$



$$= \frac{(7\sqrt{5}+15)(\sqrt{5}-2)}{4} l_{20}^3 = \frac{35+15\sqrt{5}-14\sqrt{5}-30}{4} l_{20}^3 = \boxed{\frac{5+\sqrt{5}}{4} l_{20}^3}$$

- 2) Las 12 pirámides complementarias tienen por base un pentágono regular de lado igual a  $\frac{l_{20}}{2}$ , y las caras laterales formadas por 5 triángulos equiláteros también de lado  $\frac{l_{20}}{2}$ ; el radio de la circunferencia circunscrita al pentágono de la base, se obtiene de la fórmula de geometría métrica

$$l_5 = \frac{R}{2} \sqrt{10-2\sqrt{5}}$$

en la que despejando  $R$  y substituyendo  $l_5$  por  $\frac{l_{20}}{2}$ , nos dará

$$R = \frac{2 l_5}{\sqrt{10-2\sqrt{5}}} = \frac{1}{2} \sqrt{\frac{5+\sqrt{5}}{10}} l_{20}$$

Desarrollo del cálculo anterior:  $R = \frac{2 l_5}{\sqrt{10-2\sqrt{5}}} = \sqrt{\frac{4}{10-2\sqrt{5}}} l_5 =$

$$= \sqrt{\frac{2}{5-\sqrt{5}}} l_5 = \sqrt{\frac{2(5+\sqrt{5})}{20}} l_5 = \sqrt{\frac{5+\sqrt{5}}{10}} \times \frac{l_{20}}{2} = \boxed{\frac{1}{2} \sqrt{\frac{5+\sqrt{5}}{10}} l_{20}}$$

La altura  $h$  de una de las pirámides anteriores, la (sigue y concluye este párrafo en el reverso)

$$h = \sqrt{\left(\frac{l_{20}}{2}\right)^2 - R^2} = \sqrt{\left(\frac{l_{20}}{2}\right)^2 - \left(\frac{1}{2} \sqrt{\frac{5+\sqrt{5}}{10}} l_{20}\right)^2} = \frac{1}{2} \sqrt{\frac{5-\sqrt{5}}{10}} l_{20}$$

= Desarrollo del cálculo anterior:  $h = \sqrt{\left(\frac{l_{20}}{2}\right)^2 - \left(\frac{1}{2} \sqrt{\frac{5+\sqrt{5}}{10}} l_{20}\right)^2} =$

$$f(x) = \frac{1}{x^2} = x^{-2}$$

Let  $f(x) = \frac{1}{x^2}$  be a function defined for  $x \neq 0$ .  
 We want to find the derivative of  $f(x)$  using the power rule.  
 The power rule states that if  $f(x) = x^n$ , then  $f'(x) = nx^{n-1}$ .  
 In this case,  $n = -2$ . So, we have  $f'(x) = -2x^{-2-1}$ .  
 Simplifying, we get  $f'(x) = -2x^{-3}$ .  
 This can be written as  $f'(x) = -\frac{2}{x^3}$ .

$$f'(x) = -\frac{2}{x^3}$$

Therefore, the derivative of  $f(x) = \frac{1}{x^2}$  is  $f'(x) = -\frac{2}{x^3}$ .

$$f'(x) = -\frac{2}{x^3}$$

Let's verify this result using the definition of the derivative.

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \rightarrow 0} \frac{\frac{1}{(x+h)^2} - \frac{1}{x^2}}{h}$$

First, we find a common denominator for the numerator:

$$\frac{1}{(x+h)^2} - \frac{1}{x^2} = \frac{x^2 - (x+h)^2}{x^2(x+h)^2}$$

Now, we substitute this back into the limit expression:

$$= \sqrt{\frac{1}{4} - \frac{1}{4} \times \frac{5+\sqrt{5}}{10}} \quad l_{20} = \frac{1}{2} \sqrt{1 - \frac{5+\sqrt{5}}{10}} \quad l_{20} = \boxed{\frac{1}{2} \sqrt{\frac{5-\sqrt{5}}{10}} \quad l_{20}}$$

El área de la base pentagonal, será (ver geometría métrica)

$$S_5 = \frac{\sqrt{25+10\sqrt{5}}}{4} \times \left(\frac{l_{20}}{2}\right)^2$$

y el volumen de las 12 pirámides pentagonales

$$V_2 = 12 \times \frac{\sqrt{25+10\sqrt{5}}}{4} \times \left(\frac{l_{20}}{2}\right)^2 \times \frac{1}{3} \left(\frac{1}{2} \sqrt{\frac{5-\sqrt{5}}{10}} \quad l_{20}\right) = \frac{5+\sqrt{5}}{16} \quad l_{20}^3$$

Desarrollo del cálculo anterior:

$$V_2 = 12 \times \frac{\sqrt{25+10\sqrt{5}}}{4} \times \left(\frac{l_{20}}{2}\right)^2 \times \frac{1}{3} \left(\frac{1}{2} \sqrt{\frac{5-\sqrt{5}}{10}} \quad l_{20}\right) =$$

$$= 3 \sqrt{25+10\sqrt{5}} \times \frac{1}{4} \times \frac{1}{6} \times \sqrt{\frac{5-\sqrt{5}}{10}} \quad l_{20}^3 = \frac{1}{8} \sqrt{\frac{(25+10\sqrt{5})(5-\sqrt{5})}{10}} \quad l_{20}^3 =$$

$$= \frac{1}{8} \sqrt{\frac{125+50\sqrt{5}-25\sqrt{5}-50}{10}} \quad l_{20}^3 = \frac{1}{8} \sqrt{\frac{75+25\sqrt{5}}{10}} \quad l_{20}^3 = \frac{1}{8} \sqrt{\frac{15+5\sqrt{5}}{2}} \quad l_{20}^3 =$$

$$= \frac{1}{8} \times \frac{\sqrt{15+5\sqrt{5}}}{\sqrt{2}} \quad l_{20}^3 = \frac{1}{8} \times \frac{\sqrt{\frac{25}{2}} + \sqrt{\frac{5}{2}}}{\sqrt{2}} \quad l_{20}^3 = \frac{1}{8} \left(\sqrt{\frac{25}{4}} + \sqrt{\frac{5}{4}}\right) \quad l_{20}^3 =$$

$$= \frac{1}{8} \times \left(\frac{5}{2} + \frac{\sqrt{5}}{2}\right) \quad l_{20}^3 = \frac{1}{8} \times \frac{5+\sqrt{5}}{2} \quad l_{20}^3 = \boxed{\frac{5+\sqrt{5}}{16} \quad l_{20}^3} \quad (\text{comprobado})$$

El volumen total del poliedro, será pues

$$V = V_1 + V_2 = \frac{5+\sqrt{5}}{4} \quad l_{20}^3 + \frac{5+\sqrt{5}}{16} \quad l_{20}^3 = \frac{5(5+\sqrt{5})}{16} \quad l_{20}^3 \quad [4]$$

1.  $\frac{1}{2} \log \frac{1}{2} = -\frac{1}{2} \log 2$   
 $\frac{1}{2} \log \frac{1}{2} = -\frac{1}{2} \log 2$   
 $\frac{1}{2} \log \frac{1}{2} = -\frac{1}{2} \log 2$   
 $\frac{1}{2} \log \frac{1}{2} = -\frac{1}{2} \log 2$

2.  $\frac{1}{2} \log \frac{1}{2} = -\frac{1}{2} \log 2$   
 $\frac{1}{2} \log \frac{1}{2} = -\frac{1}{2} \log 2$   
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3.  $\frac{1}{2} \log \frac{1}{2} = -\frac{1}{2} \log 2$   
 $\frac{1}{2} \log \frac{1}{2} = -\frac{1}{2} \log 2$   
 $\frac{1}{2} \log \frac{1}{2} = -\frac{1}{2} \log 2$   
 $\frac{1}{2} \log \frac{1}{2} = -\frac{1}{2} \log 2$

llegaríamos al mismo resultado considerando como primer sumando el volumen del icosaedro regular y como segundo sumando las 20 pirámides triangulares procedentes de los ángulos sólidos del dodecaedro.

Los cálculos justificativos son los siguientes:

- 1) Volumen del icosaedro

$$V_{20} = \frac{15 + 5\sqrt{5}}{12} l_{20}^3 \quad (\text{ver lám. 4, fórm. 11), } 2$$

- 2) Volumen de las 20 pirámides triangulares, cuya base es un triángulo equilátero de lado  $\frac{l_{20}}{2}$  y altura el cateto de un triángulo rectángulo de hipotenusa  $\frac{l_{20}}{2}$  y radio  $R$  de la circunferencia circunscrita al triángulo de la base.

Sabiendo que, según se estudia en Geometría métrica, el área de un triángulo equilátero de lado  $\frac{l_{20}}{2}$ , es de

$$\frac{\sqrt{3}}{4} \cdot \left(\frac{l_{20}}{2}\right)^2 = \frac{\sqrt{3}}{16} l_{20}^2$$

y el radio  $R$  de la circunferencia circunscrita a dicha base es de

$$R = \frac{\sqrt{3}}{3} \cdot \frac{l_{20}}{2} = \frac{\sqrt{3}}{6} l_{20}$$

calcularemos la altura  $h$  de una pirámide triangular

$$h = \sqrt{\left(\frac{l_{20}}{2}\right)^2 - \left(\frac{\sqrt{3}}{6} l_{20}\right)^2} = \sqrt{\frac{1}{4} l_{20}^2 - \frac{1}{12} l_{20}^2}$$

y sustituyendo  $l_{20}$  en función de  $l_{30}$  (ver lám. 17, fórm. 181)





tendremos que

$$h = \sqrt{\frac{1}{4} \left( \frac{\sqrt{5}-1}{2} l_{20} \right)^2 - \frac{1}{12} l_{20}^2} = \frac{3-\sqrt{5}}{4\sqrt{3}} l_{20}$$

Desarrollo del cálculo anterior:  $h = \sqrt{\frac{1}{4} \left( \frac{\sqrt{5}-1}{2} l_{20} \right)^2 - \frac{1}{12} l_{20}^2} =$

$$= \frac{1}{2} \sqrt{\left( \frac{\sqrt{5}-1}{2} \right)^2 l_{20}^2 - \frac{1}{3} l_{20}^2} = \frac{1}{2} \times \sqrt{\frac{6-2\sqrt{5}}{4} - \frac{1}{3}} \times l_{20} =$$

$$= \frac{1}{2} \sqrt{\frac{3-\sqrt{5}}{2} - \frac{1}{3}} l_{20} = \frac{1}{2} \sqrt{\frac{9-3\sqrt{5}-2}{6}} l_{20} = \frac{1}{2} \sqrt{\frac{7-3\sqrt{5}}{6}} l_{20} =$$

$$= \frac{1}{2} \times \frac{\sqrt{7-3\sqrt{5}}}{\sqrt{6}} l_{20} = \frac{1}{2} \times \frac{\sqrt{\frac{9}{2}} - \sqrt{\frac{5}{2}}}{\sqrt{6}} l_{20} = \frac{1}{2} \left( \sqrt{\frac{9}{12}} - \sqrt{\frac{5}{12}} \right) l_{20} =$$

$$= \frac{1}{2} \left( \frac{3}{2\sqrt{3}} - \frac{\sqrt{5}}{2\sqrt{3}} \right) l_{20} = \frac{1}{2} \left( \frac{3-\sqrt{5}}{2\sqrt{3}} \right) l_{20} = \boxed{\frac{3-\sqrt{5}}{4\sqrt{3}} l_{20}}$$

El volumen  $V_2$  de las 20 pirámides, valdrá pues:

$$V_2 = 20 \times \frac{\sqrt{3}}{16} l_{20}^2 \times \frac{1}{3} \left( \frac{3-\sqrt{5}}{4\sqrt{3}} l_{20} \right) = \frac{20 \times \sqrt{3} \times (3-\sqrt{5})}{16 \times 3 \times 4\sqrt{3}} l_{20}^3 = \frac{5(3-\sqrt{5})}{48} l_{20}^3$$

y el volumen total del poliedro

$$V = V_{20} + V_2 = \frac{15+5\sqrt{5}}{12} l_{20}^3 + \frac{5(3-\sqrt{5})}{48} l_{20}^3 = \frac{5(5+\sqrt{5})}{16} l_{20}^3 \quad [4]$$

igual al resultado anteriormente obtenido.

Desarrollo del cálculo anterior:  $V = V_{20} + V_2 =$

$$= \frac{15+5\sqrt{5}}{12} l_{20}^3 + \frac{5(3-\sqrt{5})}{48} l_{20}^3 = \frac{60+20\sqrt{5}+15-5\sqrt{5}}{48} l_{20}^3 =$$

Q.1. Find the value of  $\sqrt{16} + \sqrt{9}$ .

Sol. We know that  $\sqrt{16} = 4$  and  $\sqrt{9} = 3$ .

$$\therefore \sqrt{16} + \sqrt{9} = 4 + 3 = 7$$

Q.2. Simplify  $\sqrt{18} + \sqrt{50}$ .

$$\sqrt{18} = \sqrt{9 \times 2} = 3\sqrt{2}$$

$$\sqrt{50} = \sqrt{25 \times 2} = 5\sqrt{2}$$

$$\therefore \sqrt{18} + \sqrt{50} = 3\sqrt{2} + 5\sqrt{2} = 8\sqrt{2}$$

Q.3. Find the value of  $\sqrt{25} - \sqrt{16}$ .

$$\sqrt{25} = 5 \text{ and } \sqrt{16} = 4$$

$$\therefore \sqrt{25} - \sqrt{16} = 5 - 4 = 1$$

Q.4. Simplify  $\sqrt{72} - \sqrt{18}$ .

$$\sqrt{72} = \sqrt{36 \times 2} = 6\sqrt{2}$$

$$\sqrt{18} = \sqrt{9 \times 2} = 3\sqrt{2}$$

$$\therefore \sqrt{72} - \sqrt{18} = 6\sqrt{2} - 3\sqrt{2} = 3\sqrt{2}$$

Q.5. Find the value of  $\sqrt{49} + \sqrt{36}$ .

$$\sqrt{49} = 7 \text{ and } \sqrt{36} = 6$$

$$\therefore \sqrt{49} + \sqrt{36} = 7 + 6 = 13$$

$$= \frac{75 + 15\sqrt{5}}{48} l_{20}^3 = \frac{25 + 5\sqrt{5}}{16} l_{20}^3 = \boxed{\frac{5(5 + \sqrt{5})}{16} l_{20}^3}$$

V = Volumen en función de  $l_{12}$

Siendo  $l_{20} = \frac{\sqrt{5} + 1}{2} l_{12}$  (ver lám. 17, fórm. 194),

substituyendo este valor en [4], tendremos:

$$V = \frac{5(5 + \sqrt{5})}{16} l_{20}^3 = \frac{5(5 + \sqrt{5})}{16} \left( \frac{\sqrt{5} + 1}{2} l_{12} \right)^3 = \frac{5 \cdot (15 + 7\sqrt{5})}{16} l_{12}^3$$

Desarrollo del cálculo anterior  $V = \frac{5(5 + \sqrt{5})}{16} \times \left( \frac{\sqrt{5} + 1}{2} l_{12} \right)^3 =$

$$= \frac{5(5 + \sqrt{5})}{16} \times \frac{5\sqrt{5} + 15 + 3\sqrt{5} + 1}{8} l_{12}^3 = \frac{5(5 + \sqrt{5})}{16} \times \frac{16 + 8\sqrt{5}}{8} l_{12}^3 =$$

$$= \frac{5(5 + \sqrt{5})}{16} \times (2 + \sqrt{5}) l_{12}^3 = \frac{5(10 + 2\sqrt{5} + 5\sqrt{5} + 5)}{16} l_{12}^3 = \boxed{\frac{5(15 + 7\sqrt{5})}{16} l_{12}^3}$$

Resumen de resultados

### CUADRO SINÓPTICO

Magnitud	Valor exacto	Valor decimal aproximado
$\alpha$		197
S	$\frac{15(\sqrt{3} + \sqrt{5} - 2\sqrt{2})}{4} l_{20}^2$	9, 21 97 25... $l_{20}^2$ 198
	$\frac{15(3\sqrt{3} + \sqrt{5} + \sqrt{10} + 2\sqrt{2})}{8} l_{12}^2$	24, 13 75 53... $l_{12}^2$ 199
V	$\frac{5(5 + \sqrt{5})}{16} l_{20}^3$	2, 36 12 71... $l_{20}^3$ 200
	$\frac{5(15 + 7\sqrt{5})}{16} l_{12}^3$	9, 57 88 99... $l_{12}^3$ 201



المادة: الفيزياء  
 الفصل: الميكانيكا  
 الموضوع: الحركة المتسارطة

الهدف من هذا الموضوع:

1- فهم مفهوم التسارع وكيف يتم حسابه.  
 2- التعرف على أنواع التسارع المختلفة.

3- التعرف على الفرق بين السرعة والتسارع.

4- التعرف على الفرق بين السرعة المتوسطة والسرعة اللحظية.

5- التعرف على الفرق بين التسارع المتوسط والتسارع اللحظي.

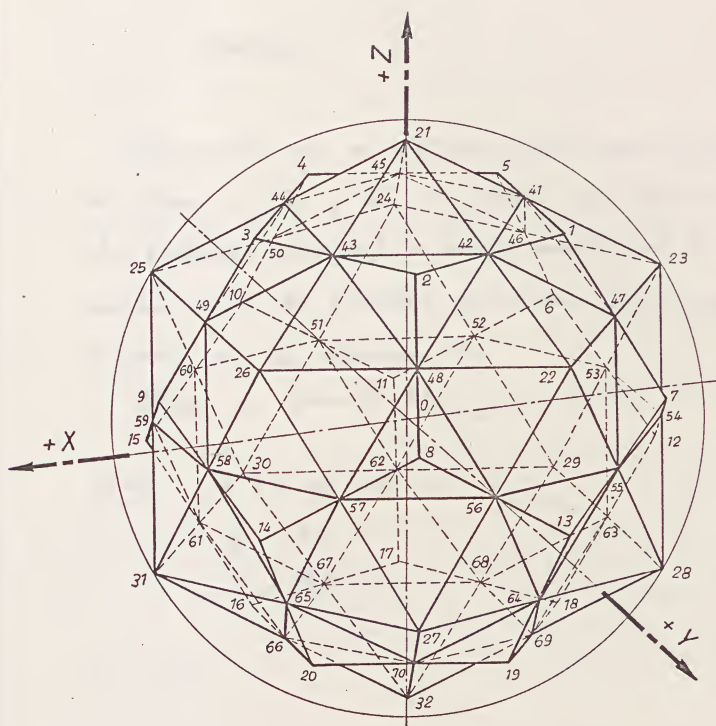
6- التعرف على الفرق بين التسارع المتوسط والتسارع اللحظي.



الاسم: \_\_\_\_\_

الوقت	المكان	الدرجة
1	2	3
4	5	6
7	8	9
10	11	12





*Poliedros regulares convexos conjugados*



THE UNIVERSITY OF CHICAGO PRESS

## ENUNCIADO

Representar por el método gráfico-analítico, en los planos I, II y III, el poliedro que forma el núcleo (octaedro regular) o sólido común, en la intersección de los dos tetraedros regulares conjugados estudiados en la lámina 12.

Dibujar en formato A3v y a escala 1:1

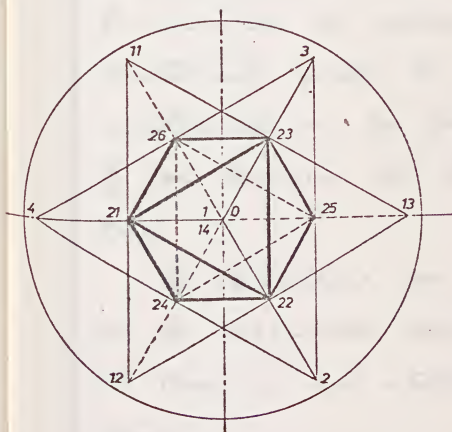
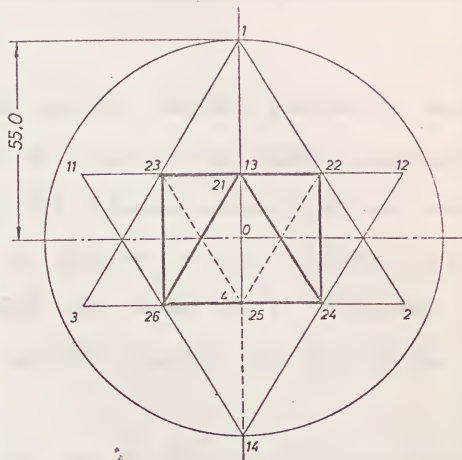
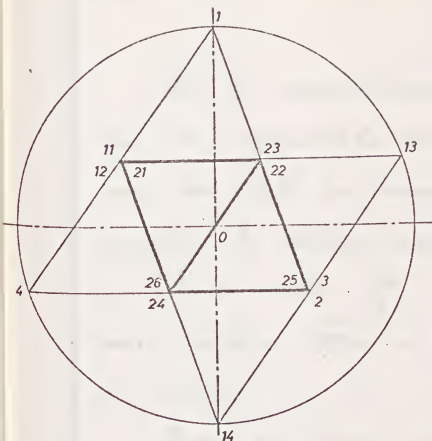
DATOS       $O (72, 72, 85) \text{ mm}$

$$l_g = 89,8 \text{ mm}$$

$$l'_h = 89,8 \text{ mm}$$

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# NUMERACIÓN DE VÉRTICES

Tetraedro dato ..... 1 al 4

Tetraedro conjugado ..... 11 al 14

Núcleo (octaedro regular) ..... 21 al 26

## ENUNCIADO

Representar por el método gráfico-analítico, en los planos I II y III, el poliedro que forma el núcleo (octaedro regular) o sólido común en la intersección de los dos poliedros conjugados (tetraedros regulares) estudiados en la lámina 12.

Dibujar en formato A3v y a escala 1:1.

Propuesta	De entrega	Entregada	Calificación	(firma)	Escuela
Fecha:					Curso
Alumno:					
Escala	Poliedros regulares convexos conjugados				Lámina 19
1:1					Curso '19 - '18





THESE TWO FIGURES ARE  
 THE SAME AS THE PREVIOUS  
 ONE, BUT THE LINES ARE  
 DRAWN IN A DIFFERENT  
 MANNER, SO AS TO SHOW  
 THE SAME PRINCIPLE IN  
 A DIFFERENT MANNER.



THESE TWO FIGURES ARE

THE SAME AS THE PREVIOUS  
 ONE, BUT THE LINES ARE  
 DRAWN IN A DIFFERENT  
 MANNER, SO AS TO SHOW  
 THE SAME PRINCIPLE IN  
 A DIFFERENT MANNER.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100
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Para la representación del poliedro pedido, tomaremos como base el representado en la lámina 12, del cual suprimiremos las partes no comunes del tetraedro dado y de su conjugado. El sólido común a ambos es un octaedro regular de lado  $\frac{l_4}{2}$  mitad del lado de los tetraedros, cuyos vértices están en los puntos medios de las aristas de ambos.

Como ya hemos estudiado en la lámina 3 el octaedro regular, omitiremos el estudio analítico de las dimensiones del núcleo, que pueden obtenerse de inmediato aplicando los valores del cuadro sinóptico final, bien directamente previa la determinación de  $l_8 = \frac{l_4}{2}$ , o bien sustituyendo en las fórmulas 21 al 28 el valor de  $l_8$  en función del lado  $l_4$  de los tetraedros conjugados.

En este último caso, y conservando la nomenclatura de las magnitudes estudiadas en la lámina 3, damos al final de este estudio el cuadro sinóptico correspondiente.

La posición del octaedro núcleo en la lámina 19, queda alterada con respecto a la que figura en la lámina 3, teniendo en aquella una cara paralela a  $\Pi_2$  la contigua perpendicular a  $\Pi_1$ .



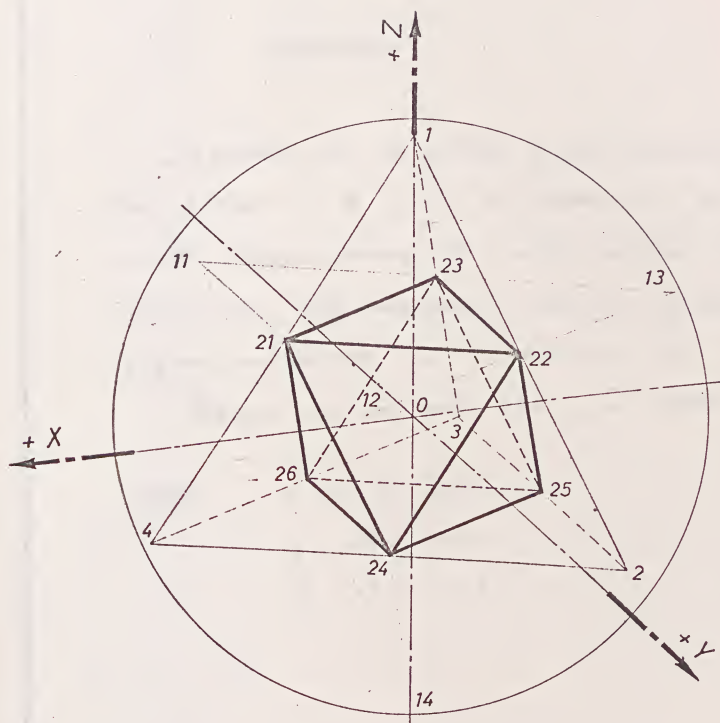
## CUADRO SINÓPTICO

Magnitud	Valor exacto	Valor decimal aproximado
$a_8$	<sup>202</sup> $\frac{\sqrt{2}}{4} l_4$	0,35 35 53 ... $l_4$
$b_8$	<sup>203</sup> $\frac{1}{4} l_4$	0,25 00 00 ... $l_4$
$c_8$	<sup>204</sup> $\frac{\sqrt{6}}{12} l_4$	0,20 41 24 ---- $l_4$
$d_8$	<sup>205</sup> $\frac{\sqrt{3}}{6} l_4$	0,28 86 75 ---- $l_4$
$2\psi_8$	<sup>206</sup> $\text{sen } \psi = \frac{\sqrt{6}}{3}$	
$k_8$	<sup>207</sup> $\frac{\sqrt{3}}{12} l_4$	0,14 43 38 ---- $l_4$
$S_8$	<sup>208</sup> $\frac{\sqrt{3}}{2} l_4^2$	0,86 60 25 ---- $l_4^2$
$V_8$	<sup>209</sup> $\frac{\sqrt{2}}{24} l_4^3$	0,05 89 26 ---- $l_4^3$
Relaciones entre magnitudes		
<sup>210</sup>	$d_8 = 2 k_8$	$l_8 = 2 b_8$



Account	Debit	Credit
Jan 1 Balance		100.00
Jan 2	10.00	
Jan 3		5.00
Jan 4	2.00	
Jan 5		1.00
Jan 6	1.00	
Jan 7		1.00
Jan 8	1.00	
Jan 9		1.00
Jan 10	1.00	
Jan 11		1.00
Jan 12	1.00	
Jan 13		1.00
Jan 14	1.00	
Jan 15		1.00
Jan 16	1.00	
Jan 17		1.00
Jan 18	1.00	
Jan 19		1.00
Jan 20	1.00	
Jan 21		1.00
Jan 22	1.00	
Jan 23		1.00
Jan 24	1.00	
Jan 25		1.00
Jan 26	1.00	
Jan 27		1.00
Jan 28	1.00	
Jan 29		1.00
Jan 30	1.00	
Jan 31		1.00
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Feb 31	1.00	
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Mar 3		1.00
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Mar 7		1.00
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Mar 9		1.00
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Mar 15		1.00
Mar 16	1.00	
Mar 17		1.00
Mar 18	1.00	
Mar 19		1.00
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Poliedros regulares convexos conjugados



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## B) Centros de aristas

## ENUNCIADO

Representar por el método gráfico-analítico, en los planos I, II y III, el poliedro que forma el núcleo (arquimédico III) o sólido común, en la intersección del exaedro y octaedro regulares conjugados estudiados en la lámina 15.

Dibujar en formato A3V y a escala 1:1.

DATOS

$$O (72, 72, 85) \text{ mm}$$

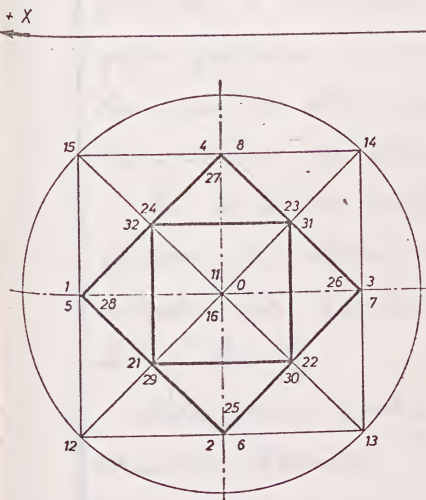
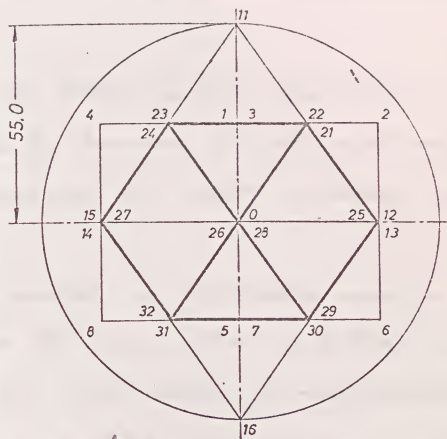
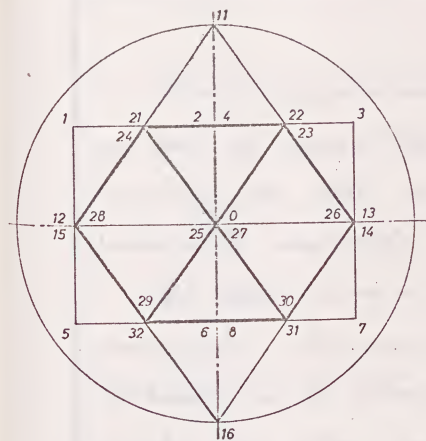
$$l_8 = 55,0 \text{ mm}$$

$$l'_8 = 77,8 \text{ mm}$$

# CONTENTS

Original Articles  
 The Effect of the Diet on the Blood Sugar in the Normal Individual  
 The Effect of the Diet on the Blood Sugar in the Diabetic Individual  
 The Effect of the Diet on the Blood Sugar in the Obese Individual  
 The Effect of the Diet on the Blood Sugar in the Thin Individual  
 The Effect of the Diet on the Blood Sugar in the Elderly Individual  
 The Effect of the Diet on the Blood Sugar in the Young Individual  
 The Effect of the Diet on the Blood Sugar in the Middle-aged Individual  
 The Effect of the Diet on the Blood Sugar in the Infirmed Individual  
 The Effect of the Diet on the Blood Sugar in the Healthy Individual  
 The Effect of the Diet on the Blood Sugar in the Sick Individual  
 The Effect of the Diet on the Blood Sugar in the Well Individual  
 The Effect of the Diet on the Blood Sugar in the Unwell Individual  
 The Effect of the Diet on the Blood Sugar in the Ailing Individual  
 The Effect of the Diet on the Blood Sugar in the Recovering Individual  
 The Effect of the Diet on the Blood Sugar in the Convalescing Individual  
 The Effect of the Diet on the Blood Sugar in the Discharged Individual  
 The Effect of the Diet on the Blood Sugar in the Deceased Individual

1924  
 May 1, 1924  
 May 1, 1924



### ENUNCIADO

Representar por el método gráfico-analítico, en los planos I, II y III, el poliedro que forma el núcleo (arquimediano III) o sólido común, en la intersección del exaedro y octaedro regulares conjugados estudiados en la lámina 15.

Dibujar en formato A3v y a escala 1:1.

#### NUMERACIÓN DE VÉRTICES

Exaedro..... 1 al 8  
Octaedro..... 11 al 16  
Núcleo (arquimediano III)..... 21 al 32

	Propuesta	De entrega	Entregada	Calificación	(firma)	Escuela
Fecha:						
Alumno:						Curso
Escala	Poliedros regulares convexos conjugados					Lámina 20
1:1						Curso 19 - 19





Para la representación del poliedro pedido, tomaremos como base el representado en la lámina 15, del cual suprimiremos las partes no comunes de ambos poliedros recíprocamente conjugados.

El sólido común a ambos, es el poliedro convexo denominado "Arquimediano III" cuyo estudio detallado se ha efectuado en la lámina 35. Las aristas de este poliedro unidos son todas iguales y el total de sus caras se compone de 8 triángulos equiláteros y 6 cuadrados. Los ángulos poliedros (12 en total) son todos iguales, y en cada uno de ellos concurren dos caras cuadradas y dos triangulares, en forma alternada.

En el cuadro sinóptico de la mencionada lámina 35, figuran los valores analíticos de las principales magnitudes del Arquimediano III, en función de su lado " $l = l_{III}$ ".

Dichos valores pueden aplicarse a este ejercicio, si previamente obtenemos " $l_{III}$ " en función de " $l_8$ " (lado del octaedro) o de " $l_6$ " (lado del exaedro), que independientemente pueden ser datos de esta representación.

Las relaciones correspondientes se deducen de esta lámina 20, en la que

a) En función del lado " $l_8$ " del octaedro

$$l_{III} = \frac{1}{2} l_8 = 0,500000... l_8 \quad (211) \quad [1]$$

The following is a list of the names of the members of the American Medical Association who have been elected to the office of President for the year 1917.

The President of the American Medical Association for the year 1917 is Dr. J. C. Brainerd, of Chicago, Ill. He was elected at the annual meeting of the Association held at the Hotel Waldorf-Astoria, New York, N. Y., on June 12, 1916. Dr. Brainerd is a member of the American Medical Association since 1885, and has served in various capacities, including President of the Illinois Medical Association, President of the American Medical Association for the year 1915, and President of the American Medical Association for the year 1916.

Dr. Brainerd is a graduate of the University of Chicago, where he received his M.D. degree in 1885. He has been a member of the American Medical Association since 1885, and has served in various capacities, including President of the Illinois Medical Association, President of the American Medical Association for the year 1915, and President of the American Medical Association for the year 1916.

Dr. Brainerd is a member of the American Medical Association since 1885, and has served in various capacities, including President of the Illinois Medical Association, President of the American Medical Association for the year 1915, and President of the American Medical Association for the year 1916.

Dr. Brainerd is a member of the American Medical Association since 1885, and has served in various capacities, including President of the Illinois Medical Association, President of the American Medical Association for the year 1915, and President of the American Medical Association for the year 1916.

b) En función del lado " $l_6$ " del exaedro conjugado

De la fórmula 148, lám. 14, en la que

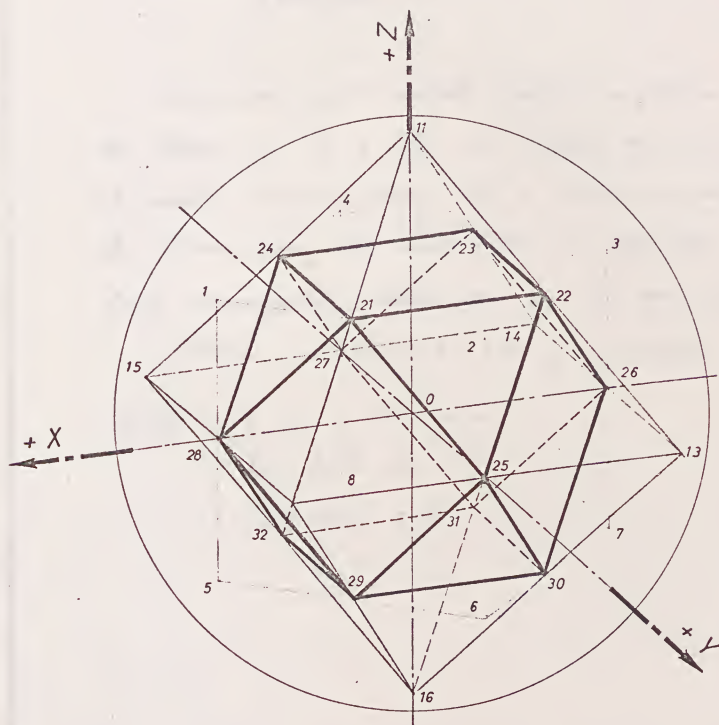
$$l_6 = \frac{\sqrt{2}}{2} l_8, \quad \text{se deduce} \quad l_8 = \frac{2}{\sqrt{2}} l_6 = \sqrt{2} l_6;$$

valor que sustituido en [1], nos da

$$\boxed{l_{III}} = \frac{1}{2} l_8 = \frac{1}{2} \sqrt{2} l_6 = \boxed{\frac{\sqrt{2}}{2} l_6} = 0.707107... l_6 \quad (212) [2]$$







*Poliedros regulares convexos conjugados*



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## ENUNCIADO

Representar por el método gráfico-analítico, en los planos I, II y III, el poliedro que forma el núcleo (arquimediano IV) o sólido común, en la intersección del dodecaedro e icosaedro regulares conjugados estudiados en la lámina 18.

Dibujar en formato A3 y a escala 1:1

DATOS  $O(72, 72, 85) \text{ mm}$

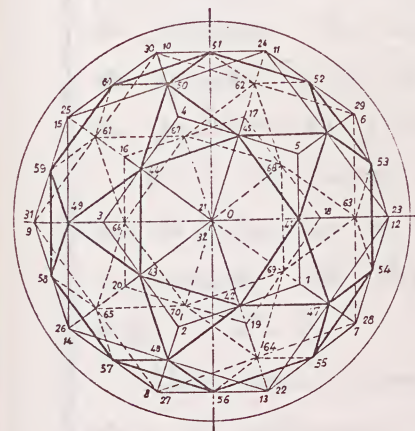
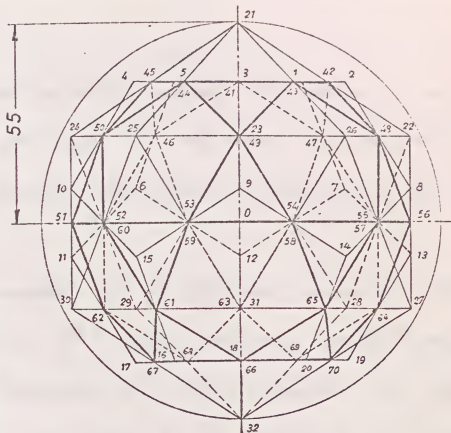
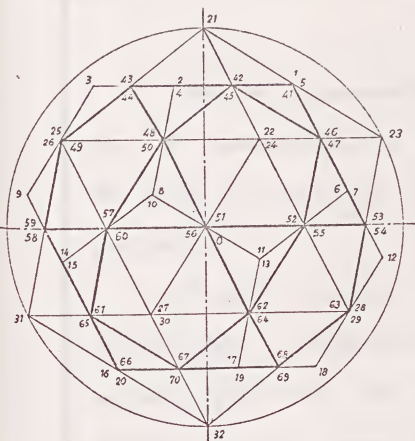
$l_{12} = 35,7 \text{ mm}$

$l'_{20} = 57,8 \text{ mm}$

ARTICLE

On the question of the origin of the human race, the evidence is so conflicting that it is difficult to arrive at a definite conclusion. The various theories advanced by different writers are based on different assumptions, and the results are accordingly different. Some writers maintain that the human race is descended from a single pair of individuals, while others maintain that it is descended from many different pairs. Some writers maintain that the human race is descended from a single pair of individuals, while others maintain that it is descended from many different pairs.

THE JOURNAL OF THE  
ROYAL ANTHROPOLOGICAL INSTITUTE  
Vol. 10, Part 1



#### NUMERACIÓN DE VÉRTICES

- Dodecaedro regular..... 1 al 20  
 Icosaedro regular..... 21 al 32  
 Núcleo (arquimediano IV)..... 41 al 70

#### ENUNCIADO

Representar por el método gráfico-analítico, en los planos I, II y III el poliedro que forma el núcleo (arquimediano IV) o sólido común, en la intersección del dodecaedro e icosaedro regulares conjugados estudiados en la lámina 18.

Dibujar en formato A3v y a escala 1:1.

Propuesta	De entrega	Entregada	Calificación	(firma)	Escuela
Fecha:					Curso
Alumno:					
Escala	Poliedros regulares convexos conjugados				Lámina 21
1:1					Curso 19 - 19





Diagram 1

The following diagram illustrates the process of the human mind in the state of deep meditation. It shows the flow of energy from the base of the spine, through the chakras, and out through the crown of the head. The diagram is divided into three main sections: the base, the middle, and the top. The base section shows the energy entering the body through the feet. The middle section shows the energy moving through the chakras, which are represented by colored circles. The top section shows the energy exiting the body through the crown of the head. The diagram is a simplified representation of the human energy system.

Diagram 2



Diagram 3

The following diagram illustrates the process of the human mind in the state of deep meditation. It shows the flow of energy from the base of the spine, through the chakras, and out through the crown of the head. The diagram is divided into three main sections: the base, the middle, and the top. The base section shows the energy entering the body through the feet. The middle section shows the energy moving through the chakras, which are represented by colored circles. The top section shows the energy exiting the body through the crown of the head. The diagram is a simplified representation of the human energy system.

Para la representación del poliedro pedido, tomaremos como base el representado en la lámina 18, del cual suprimiremos las partes no comunes de ambos poliedros recíprocamente conjugados.

El sólido común a ambos es el poliedro convexo denominado "Arquimedeano IV" cuyo estudio detallado se ha efectuado en la lámina 36. Las aristas de este poliedro todas son todas iguales y el total de sus caras se compone de 12 pentágonos regulares y 20 triángulos equiláteros de lados de igual longitud en ambos polígonos. Los ángulos poliedros (30 en total) son todos iguales y en cada uno de ellos concurren dos caras pentagonales y dos triangulares en forma alternada.

En los dos cuadros sinópticos de la mencionada lámina 36, figuran los valores analíticos de las principales magnitudes del Arquimedeano IV, en función de su lado " $l = l_{IV}$ ".

Dichos valores pueden aplicarse a este ejercicio, si previamente obtenemos " $l_{IV}$ " en función " $l_{20}$ " (lado del icosaedro) o de " $l_{12}$ " (lado del dodecaedro), que independientemente pueden ser datos de esta representación.

Las relaciones correspondientes se deducen de esta lámina 21, en la que



a) En función del lado " $l_{20}$ " del icosaedro

$$\boxed{l_{IV}} = \frac{1}{2} l_{20} = 0,50 \ 00 \ 00 \dots l_{20} \quad (213) \quad [1]$$

b) En función del lado " $l_{12}$ " del dodecaedro conjugado

De la fórmula 181, tam 17, en la que

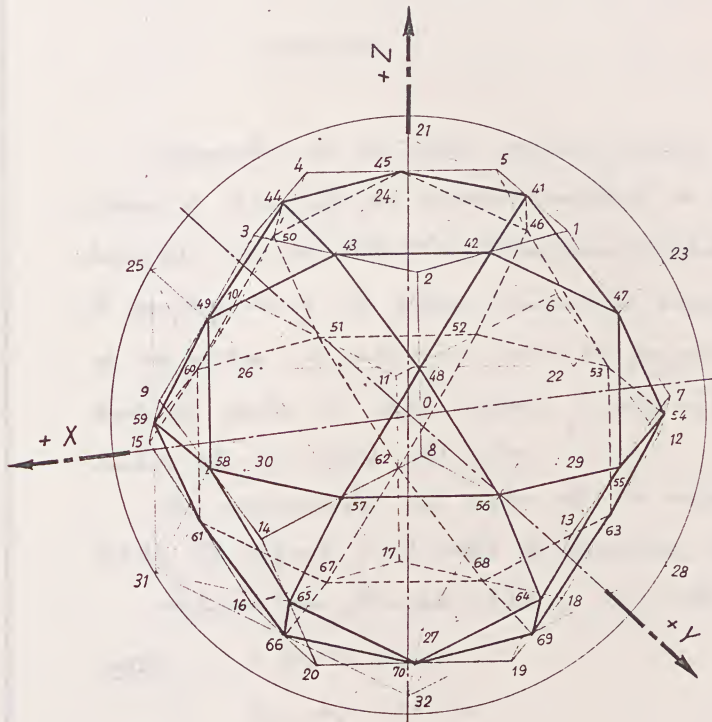
$$l_{12} = \frac{\sqrt{5}-1}{2} l_{20}, \text{ se deduce } l_{20} = \frac{2}{\sqrt{5}-1} l_{12};$$

tal que sustituido en [1], nos da

$$\begin{aligned} \boxed{l_{IV}} &= \frac{1}{2} l_{20} = \frac{1}{2} \times \frac{2}{\sqrt{5}-1} l_{12} = \frac{1}{\sqrt{5}-1} l_{12} = \frac{\sqrt{5}+1}{4} l_{12} = \\ &= 0,80 \ 90 \ 17 \ 00 \dots l_{12} \end{aligned} \quad (214) \quad [2]$$







Poliedros regulares convexos conjugados



Polymers of various kinds, including the following:

## ENUNCIADO

Representar por el método gráfico-analítico, en los planos I, II y III, el poliedro resultante de la intersección de dos tetraedros regulares recíprocamente conjugados, y de esfera circunscrita común, siendo los vértices de cada uno de ellos las proyecciones desde el centro de dicha esfera, y sobre ésta, de los centros de las caras del otro.

Las coordenadas del centro de la esfera son:  $O(72, 72, 85)$  mm y el radio de la misma de 55 mm.

Dibujar en formato A3V y a escala 1:1.

DATOS

$O(72, 72, 85)$  mm

$a_h = a'_h = 55$  mm

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CONSIDERACIONES PREVIAS

En las láminas 6 a 10, hemos estudiado los poliedros regulares conjugados de los cinco poliedros regulares, obtenidos aquéllos al unir los centros de dos caras contiguas de éstos.

En la lámina 11 indicamos la posibilidad de deducir los poliedros conjugados de los regulares, por otros procesos geométricos distintos del anterior. El proceso seguido en el desarrollo de las láminas 11, 13, 14, 16 y 17, ha sido el de trazar por los puntos medios de las aristas del poliedro dado, rectas perpendiculares al plano determinado por dichas aristas y el centro del poliedro dado.

El nuevo proceso que seguiremos en el desarrollo de las láminas 22, 23 y 24 consistirá en considerar la esfera circunscrita al poliedro regular  $P$  dado y la de su conjugado  $P'$ , de igual radio y centro (coincidentes). En este supuesto, los vértices del conjugado  $P'$  se obtienen proyectando los centros de cada cara de  $P$ , sobre la esfera común y desde el centro de ésta.

El poliedro conjugado de un tetraedro regular  $P_4$ , ya sabemos que es otro tetraedro  $P'_4$  también regular. Por tener ambos poliedros la esfera circunscrita común, dichos poliedros serán pues iguales





Como esta propiedad ha sido deducida en la lámina 11 al considerar una distinta forma de obtención del tetraedro conjugado de otro dado, llegamos a la conclusión de que el estudio realizado en la mencionada lámina 11, y la representación conjunta del poliedro resultante en la lámina 12, es aplicable al caso que nos ocupa.

La magnitud del lado  $l'_4$  del conjugado, en función del radio  $\underline{a_4}$  común para ambos poliedros, se deduce de la fórmula 126, lám. 11, en la que

$$a_4 = a'_4 = \frac{\sqrt{6}}{4} l'_4 \quad \text{de donde} \quad l'_4 = \frac{4}{\sqrt{6}} a_4 = \frac{2\sqrt{6}}{3} a_4 = (215) [1]$$

$$= 1,63 \ 29 \ 93 \dots a_4$$

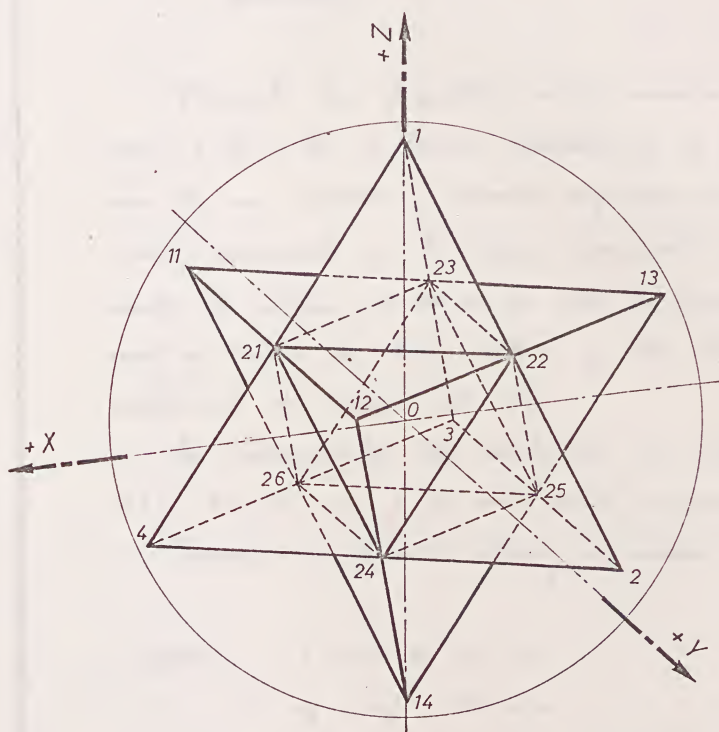
$$l'_4 = 1,63 \ 29 \ 93 \dots \times 55 = 89,8 \text{ mm.}$$

I am writing you to tell you that I am  
very well and hope you are the same.  
I have been thinking of you very much  
and hope you are all well. I have  
been very busy lately but I have  
managed to find some time to write  
to you. I hope you are all well  
and happy. I have been thinking of  
you very much and hope you are  
all well. I have been very busy  
lately but I have managed to find  
some time to write to you.

Yours truly,  
[Signature]

1914

1914



*Poliedros regulares convexos conjugados*



Figure 1. A diagram of a Sri Yantra, a complex geometric figure consisting of nine interlocking triangles that surround a central point (bindu). The triangles are arranged in a way that they form a total of 43 smaller triangles. The entire figure is enclosed within a circle, and there are additional lines extending from the vertices of the triangles to the circle's circumference.



c) De igual esfera circunscrita.

## ENUNCIADO

Representar por el método gráfico-analítico, en los planos I, II y III, el poliedro resultante de la intersección de un exaedro y octaedro regulares recíprocamente, conjugados y de esfera circunscrita común, siendo los vértices de cada uno de ellos las proyecciones desde el centro de dicha esfera, y sobre ésta, de los centros de las caras del otro.

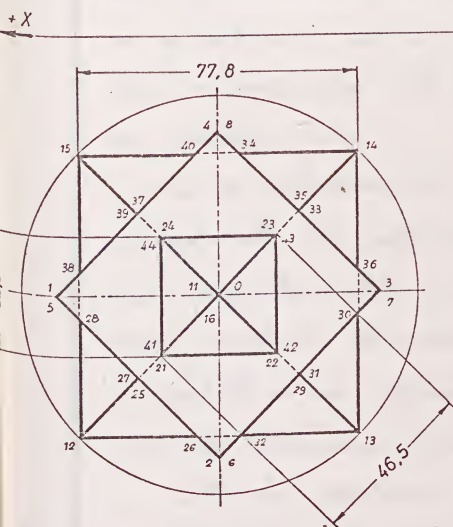
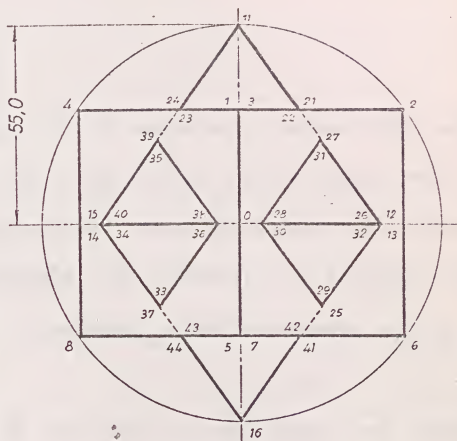
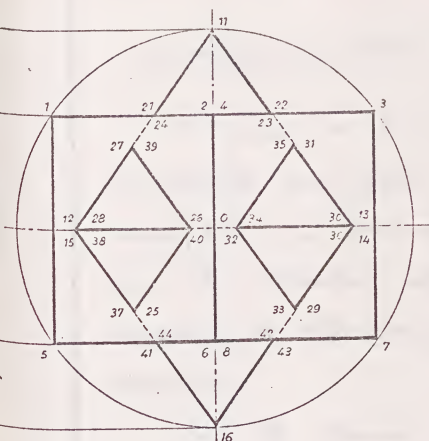
Las coordenadas del centro de la esfera son:  
 $O(72, 72, 85)$  mm y el radio de la misma de 55 mm.

Dibujar en formato A3V y a escala 1:1.

DATOS:  $O(72, 72, 85)$  mm

$a_6 = a_8 = 55$  mm.





### ENUNCIADO

Representar por el método gráfico-analítico, en los planos I II y III, el poliedro resultante de la intersección de un exaedro y octaedro regulares recíprocamente conjugados y de esfera circunscrita común, siendo los vértices de cada uno de ellos las proyecciones desde el centro de dicha esfera, y sobre ésta, de los centros de las caras del otro.

Las coordenadas del centro de la esfera son: O (72, 72, 85) mm y el radio de la misma de 55 mm.

Dibujar en formato A3v y a escala 1:1.

### NUMERACIÓN DE VÉRTICES

Exaedro ..... 1 al 8  
Octaedro ..... 11 al 16  
Puntos de intersección ..... 21 al 44

Propuesta	De entrega	Entregada	Calificación	(firma)	Escuela
Fecha:					Curso
Alumno:					
Escala	Poliedros regulares convexos conjugados				Lámina 23
1:1					Curso 12 -19



The first diagram shows a cube's net, which is a 2D representation of a 3D object. It consists of a central square with four triangles attached to its sides, forming a cross-like shape. This is a common way to represent a cube in 2D.

The second diagram shows a cube's net, which is a 2D representation of a 3D object. It consists of a central square with four triangles attached to its sides, forming a cross-like shape. This is a common way to represent a cube in 2D.

The third diagram shows a cube's net, which is a 2D representation of a 3D object. It consists of a central square with four triangles attached to its sides, forming a cross-like shape. This is a common way to represent a cube in 2D.

The fourth diagram shows a cube's net, which is a 2D representation of a 3D object. It consists of a central square with four triangles attached to its sides, forming a cross-like shape. This is a common way to represent a cube in 2D.



The fifth diagram shows a cube's net, which is a 2D representation of a 3D object. It consists of a central square with four triangles attached to its sides, forming a cross-like shape. This is a common way to represent a cube in 2D.

The sixth diagram shows a cube's net, which is a 2D representation of a 3D object. It consists of a central square with four triangles attached to its sides, forming a cross-like shape. This is a common way to represent a cube in 2D.

CONSIDERACIONES PREVIAS

Al iniciar en la lámina 6 el estudio y propiedades de los poliedros conjugados de los regulares, vimos que estos conjugados son también regulares, correspondiendo al tetraedro otro tetraedro; al exaedro el octaedro; al octaedro el exaedro; al dodecaedro el icosaedro y al icosaedro el dodecaedro.

Con ellos llegamos a la conclusión de que el exaedro y octaedro son mutuamente conjugados (cada uno lo es del otro), así como igual ocurre con el dodecaedro e icosaedro. El tetraedro regular es conjugado de sí mismo.

De lo anteriormente expuesto se deduce que al estudiar el conjugado de un poliedro dado, obtenido según un proceso geométrico determinado, conoceremos de antemano la forma constante de dicho conjugado, y en general variable con respecto a sus dimensiones, dependiendo del proceso de obtención aplicado.

Ello ha quedado de manifiesto en el estudio de las láminas 6 a 10 y 11-13-14-16-17. Los procesos geométricos de generación aplicados, confirman esta variabilidad, excepción hecha del conjugado del tetraedro en la lámina 11, cuya ley de formación da lugar a un conjugado de igual magnitud que el dado.



The American Medical Association is a national organization of physicians and surgeons, organized for the purpose of promoting the science and art of medicine and surgery, and for the betterment of the human race. It is the largest and most influential of medical organizations in the United States, and its members are the leading authorities in their respective fields. The Association's primary concern is the advancement of medical knowledge and the improvement of medical practice. It achieves this through its various departments, committees, and publications. The Journal of the American Medical Association is one of its most important publications, providing physicians with the latest information on medical research, clinical practice, and public health. The Association also advocates for the interests of the medical profession and the public, and works to ensure that medical practice is based on sound scientific principles. Its efforts have led to significant improvements in medical care and the health of the American people.

El proceso seguido en el estudio de los poliedros conjugados desarrollados en las láminas 22, 23 y 24, de esfera circunscrita común, reproduce en la lámina 22 la representación de las láminas 11 y 12. En las láminas 23, 24, el poliedro resultante de la mutua intersección del poliedro dado y de su conjugado de igual esfera circunscrita, es el mismo e independiente del que se tome como dato.



### PROCESO GRÁFICO

Procederemos previamente a la representación del exaedro y octaedro conjugados, inscritos en una misma esfera de 55 mm. de radio.

Comencemos por el octaedro regular; que colocaremos en la posición representada en la lámina 3, siguiendo el proceso gráfico de la misma. El lado  $l_8$  de este octaedro, será (ver lám. 3, fórm. 21)

$$l_8 = \frac{a_8}{\frac{\sqrt{2}}{2}} = \frac{55}{0.707107} = 77.8 \text{ mm}$$

El exaedro regular conjugado del anterior e inscrito en su misma esfera, tendrá una posición en relación con el octaedro ya dibujado, exactamente igual al que tiene en la lámina 2. El lado  $l_6$  de este exaedro, será (ver lám. 2, fórm. 11)



$$l_6 = \frac{a_6}{\frac{\sqrt{3}}{2}} = \frac{55}{0.866025} = 63,5 \text{ mm.}$$

Plantando el problema de intersección de ambos poliedros, se puede obtener el resultado por aplicación del método de intersección de caras o de aristas cuyo fundamento se establece en la geometría descriptiva.

Esta intersección se reduce a la penetración de los ángulos sólidos del octaedro en las caras del exaedro que, por razones de simetría en la posición relativa de ambos poliedros, son todas iguales y cuadradas. Cambiando las proyecciones de los dos poliedros en los planos I y II, es inmediata la obtención de la quinta intersección en ambos planos de los cuales se deduce muy fácilmente la proyección en III (igual a la I).

El sólido resultante se compone básicamente de un exaedro regular en cuyas caras y centradas con ellas, se apoyan seis pirámides rectas de base cuadrada todas iguales. Las diagonales de las bases de estas pirámides son perpendiculares (o paralelas) a los lados del cuadrado de cada cara del exaedro.

#### PROCESO GRÁFICO-ANALÍTICO

Para facilitar el dibujo y exactitud en el trazado, calcularemos las siguientes magnitudes en función del radio a de la esfera circunscrita común:





$l_6$  = Lado del exaedro

$l_8$  = Lado del octaedro

$f$  = Lado del cuadrado intersección de un ángulo recto del octaedro con una cara del exaedro.

$g$  = Diagonal del cuadrado anterior

$h$  = Altura de la pirámide exterior a una cara del exaedro

$S$  = Superficie lateral

$V$  = Volumen.

$2\theta$  = Ángulo rectilíneo del diedro formado por la intersección de una cara del exaedro con la correspondiente del octaedro

Lado " $l_6$ " del exaedro

ya lo hemos determinado anteriormente; (su valor se deduce de la fórmula 11, lám. 2).

$$l_6 = \frac{a}{\frac{\sqrt{3}}{2}} = \frac{2\sqrt{3}}{3} a$$

Lado " $l_8$ " del octaedro

Iguualmente hemos visto que (lám. 3, fórm. 21)

$$l_8 = \frac{a}{\frac{\sqrt{2}}{2}} = \sqrt{2} a$$

Altura " $h$ " de la pirámide exterior a una cara del exaedro

En la lámina 23 se puede observar que dicha al-



tura es la diferencia entre el radio  $a$  de la esfera inscrita y el semilado  $\frac{l_6}{2}$  del exaedro, por lo que

$$h = a - \frac{l_6}{2} = a - \frac{2\sqrt{3}}{3} \times \frac{1}{2} a = \frac{3 - \sqrt{3}}{3} a$$

Lado "f" del cuadrado intersección de un ángulo sólido del octaedro con una cara del exaedro

En la lámina 23 podemos comprobar que las cuatro caras concurrentes en un vértice del octaedro forman una pirámide recta de base cuadrada y altura  $a$ . Esta pirámide es cortada por una cara del exaedro cuyo plano es paralelo a la base de aquella, produciendo otra pirámide recta de base cuadrada semejante a la primera.

La razón de semejanza entre estas dos pirámides será la que exista entre sus respectivas alturas, o sea

$$\frac{a}{h} = \frac{a}{\frac{3 - \sqrt{3}}{3} a} = \frac{3 + \sqrt{3}}{2} \quad [1]$$

Por consiguiente, tendremos que

$$l_8 = f \times \frac{3 + \sqrt{3}}{2} \quad \text{de donde}$$

$$f = \frac{2 l_8}{3 + \sqrt{3}} = \frac{2 \sqrt{2}}{3 + \sqrt{3}} a = \frac{3 \sqrt{2} - \sqrt{6}}{3} a$$

I am writing to you to inform you of the results of the survey conducted by the Department of Education. The survey was conducted in order to determine the needs of the students in the district. The results of the survey are as follows:

The first result is that the majority of the students in the district are in need of additional resources. This is due to the fact that the district is currently facing a budget deficit. As a result, the district is unable to provide the same level of resources as in previous years.

The second result is that the majority of the students in the district are in need of additional support. This is due to the fact that the district is currently facing a high level of student absenteeism. As a result, the district is unable to provide the same level of support as in previous years.

The third result is that the majority of the students in the district are in need of additional instruction. This is due to the fact that the district is currently facing a high level of student achievement. As a result, the district is unable to provide the same level of instruction as in previous years.

The fourth result is that the majority of the students in the district are in need of additional assessment. This is due to the fact that the district is currently facing a high level of student assessment. As a result, the district is unable to provide the same level of assessment as in previous years.

The fifth result is that the majority of the students in the district are in need of additional evaluation. This is due to the fact that the district is currently facing a high level of student evaluation. As a result, the district is unable to provide the same level of evaluation as in previous years.

The sixth result is that the majority of the students in the district are in need of additional monitoring. This is due to the fact that the district is currently facing a high level of student monitoring. As a result, the district is unable to provide the same level of monitoring as in previous years.

The seventh result is that the majority of the students in the district are in need of additional reporting. This is due to the fact that the district is currently facing a high level of student reporting. As a result, the district is unable to provide the same level of reporting as in previous years.

The eighth result is that the majority of the students in the district are in need of additional documentation. This is due to the fact that the district is currently facing a high level of student documentation. As a result, the district is unable to provide the same level of documentation as in previous years.

The ninth result is that the majority of the students in the district are in need of additional communication. This is due to the fact that the district is currently facing a high level of student communication. As a result, the district is unable to provide the same level of communication as in previous years.

The tenth result is that the majority of the students in the district are in need of additional collaboration. This is due to the fact that the district is currently facing a high level of student collaboration. As a result, the district is unable to provide the same level of collaboration as in previous years.

(1) 
$$\frac{d^2y}{dx^2} = \frac{dy}{dx} \cdot \frac{dy}{dx} = \frac{dy}{dx} \cdot \frac{dy}{dx}$$

(2) 
$$\frac{d^2y}{dx^2} = \frac{dy}{dx} \cdot \frac{dy}{dx} = \frac{dy}{dx} \cdot \frac{dy}{dx}$$

(3) 
$$\frac{d^2y}{dx^2} = \frac{dy}{dx} \cdot \frac{dy}{dx} = \frac{dy}{dx} \cdot \frac{dy}{dx}$$

(4) 
$$\frac{d^2y}{dx^2} = \frac{dy}{dx} \cdot \frac{dy}{dx} = \frac{dy}{dx} \cdot \frac{dy}{dx}$$

(5) 
$$\frac{d^2y}{dx^2} = \frac{dy}{dx} \cdot \frac{dy}{dx} = \frac{dy}{dx} \cdot \frac{dy}{dx}$$

(6) 
$$\frac{d^2y}{dx^2} = \frac{dy}{dx} \cdot \frac{dy}{dx} = \frac{dy}{dx} \cdot \frac{dy}{dx}$$

(7) 
$$\frac{d^2y}{dx^2} = \frac{dy}{dx} \cdot \frac{dy}{dx} = \frac{dy}{dx} \cdot \frac{dy}{dx}$$

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$$\frac{d^2y}{dx^2} = \frac{dy}{dx} \cdot \frac{dy}{dx} = \frac{dy}{dx} \cdot \frac{dy}{dx}$$

(9) 
$$\frac{d^2y}{dx^2} = \frac{dy}{dx} \cdot \frac{dy}{dx} = \frac{dy}{dx} \cdot \frac{dy}{dx}$$

(10) 
$$\frac{d^2y}{dx^2} = \frac{dy}{dx} \cdot \frac{dy}{dx} = \frac{dy}{dx} \cdot \frac{dy}{dx}$$

Desarrollo del cálculo anterior:  $m = \frac{2\sqrt{2}}{3+\sqrt{3}} a =$

$$= \frac{2\sqrt{2}(3-\sqrt{3})}{6} a = \boxed{\frac{3\sqrt{2}-\sqrt{6}}{3} a}$$

Diagonal "g" del cuadrado anterior

su valor será:  $g = f\sqrt{2} = \frac{3\sqrt{2}-\sqrt{6}}{3} \times \sqrt{2} a = \frac{6-2\sqrt{3}}{3} a$

Desarrollo del cálculo anterior:  $d = \frac{3\sqrt{2}-\sqrt{6}}{3} \times \sqrt{2} a =$

$$= \frac{3 \times \sqrt{2} \sqrt{2} - \sqrt{2} \sqrt{6}}{3} a = \frac{6-\sqrt{2}}{3} a = \boxed{\frac{6-2\sqrt{3}}{3} a}$$

Superficie lateral S

El área resultante de cada cara del exaedro, será

$$S_1 = l_6^2 - f^2 = \left(\frac{2\sqrt{3}}{3} a\right)^2 - \left(\frac{3\sqrt{2}-\sqrt{6}}{3} a\right)^2$$

El área lateral de cada pirámide exterior a dicha cara, será

$$S_2 = \frac{4f}{2} \times \frac{\sqrt{3}}{2} f = \sqrt{3} f^2 = \sqrt{3} \left(\frac{3\sqrt{2}-\sqrt{6}}{3} a\right)^2$$

y el área total S, será:

$$S = 6(S_1 + S_2) = 6 \left[ \left(\frac{2\sqrt{3}}{3} a\right)^2 - \left(\frac{3\sqrt{2}-\sqrt{6}}{3} a\right)^2 + \sqrt{3} \left(\frac{3\sqrt{2}-\sqrt{6}}{3} a\right)^2 \right]$$

$$= 4 \times (6\sqrt{3} - 8) a^2$$



1. Mathematik  
 2. Physik  
 3. Chemie  
 4. Biologie  
 5. Geographie  
 6. Englisch  
 7. Deutsch  
 8. Religion  
 9. Arbeitskunde  
 10. sonstige

$$\frac{1}{2} \cdot \frac{1}{3} = \frac{1}{6}$$

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 9. Arbeitskunde  
 10. sonstige

Desarrollo del cálculo anterior:

$$\begin{aligned}
 S &= 6 \times \left[ \left( \frac{2\sqrt{3}}{3} a \right)^2 - \left( \frac{3\sqrt{2}-\sqrt{6}}{3} a \right)^2 + \sqrt{3} \times \left( \frac{3\sqrt{2}-\sqrt{6}}{3} a \right)^2 \right] = 6 \times \left( \frac{12}{9} a^2 - \right. \\
 &\quad \left. - \frac{18+6-6\sqrt{12}}{9} a^2 + \sqrt{3} \times \frac{18+6-6\sqrt{12}}{9} a^2 \right) = 6 \times \left( \frac{4}{3} - \frac{24-6\sqrt{12}}{9} + \right. \\
 &\quad \left. + \frac{\sqrt{3} \times (24-6\sqrt{12})}{9} \right) a^2 = 6 \times \left( \frac{4}{3} - \frac{24-12\sqrt{3}}{9} + \frac{24\sqrt{3}-6\sqrt{36}}{9} \right) a^2 = \\
 &= \left( 8 - \frac{6 \times (8-4\sqrt{3})}{3} + \frac{6 \times (8\sqrt{3}-2 \times 6)}{3} \right) a^2 = \left( 8 - (16-8\sqrt{3}) + \right. \\
 &\quad \left. + (16\sqrt{3}-24) \right) a^2 = (8-16+8\sqrt{3}+16\sqrt{3}-24) a^2 = (24\sqrt{3}-32) a^2 = \\
 &= \boxed{4 \times (6\sqrt{3}-8) a^2}
 \end{aligned}$$

Volumen V

Bastará sumar al volumen del exaedro, el de las 6 pirámides exteriores; su valor será:

$$\begin{aligned}
 V &= \left( \frac{1}{6} \right)^3 + 6 f^2 \times \frac{h}{3} = \left( \frac{2\sqrt{3}}{3} a \right)^3 + 6 \left( \frac{3\sqrt{2}-\sqrt{6}}{3} a \right)^2 \times \left( \frac{3-\sqrt{3}}{3} \times \frac{a}{3} \right) = \\
 &= \frac{8 \times (9-4\sqrt{3})}{9} a^3
 \end{aligned}$$

Desarrollo del cálculo anterior:

$$V = \left( \frac{2\sqrt{3}}{3} a \right)^3 + 6 \times \left( \frac{3\sqrt{2}-\sqrt{6}}{3} a \right)^2 \times \frac{\frac{3-\sqrt{3}}{3} a}{3} = \frac{8 \times 3\sqrt{3}}{27} a^3 +$$

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$$\begin{aligned}
 &+ 2 \times \left( \frac{18 + 6 - 6\sqrt{12}}{9} a^2 \times \frac{3 - \sqrt{3}}{3} a \right) = \left( \frac{8\sqrt{3}}{9} + 2 \times \frac{24 - 12\sqrt{3}}{9} \times \right. \\
 &\times \left. \frac{3 - \sqrt{3}}{3} \right) a^2 = \left( \frac{8\sqrt{3}}{9} + \frac{2 \times 12 \times (2 - \sqrt{3}) \times (3 - \sqrt{3})}{27} \right) a^2 = \left( \frac{8\sqrt{3}}{9} + \right. \\
 &+ \left. \frac{2 \times 4 \times (6 - 3\sqrt{3} - 2\sqrt{3} + 3)}{9} \right) a^2 = \frac{8\sqrt{3} + 8 \times (9 - 5\sqrt{3})}{9} a^2 = \\
 &= \frac{8\sqrt{3} + 72 - 40\sqrt{3}}{9} a^2 = \frac{72 - 32\sqrt{3}}{9} a^2 = \boxed{\frac{8 \times (9 - 4\sqrt{3})}{9} a^2}
 \end{aligned}$$

Ángulo rectilíneo "20" del diedro formado por la intersección de una cara del exaedro con la correspondiente del octaedro.

De las figuras de la lámina 23 se deduce que dicho ángulo 20 es suplementario del semiángulo  $\varphi$  que forman dos caras contiguas del octaedro.

El valor conocido de  $2\varphi$  es (ver lám. 3, fórm. 25) de  $109^\circ 28' 16,6''$ , y por consiguiente será

$$2\theta = 180^\circ - \frac{1}{2} (109^\circ 28' 16,6'') = 125^\circ 15' 51,7''$$

analíticamente

$$\begin{aligned}
 \sin 2\theta &= \sin \varphi = \frac{\sqrt{6}}{3} = 0,816497 \\
 \cos 2\theta &= \text{negativo}
 \end{aligned}
 \left. \vphantom{\begin{aligned} \sin 2\theta &= \sin \varphi = \frac{\sqrt{6}}{3} = 0,816497 \\ \cos 2\theta &= \text{negativo} \end{aligned}} \right\}$$

Ángulo rectilíneo "20" del diedro del exaedro

(ver lám. 2, fórm. 15)  $2\varphi_6 = 90^\circ$   $\sin \varphi_6 = \frac{\sqrt{2}}{2}$





Ángulo rectilíneo " $2\varphi_8$ " del diedro del octaedro

(ver lám. 3, fórm. 25)  $2\varphi_8 = 109^\circ 28' 16,6''$  con  $\varphi_8 = \frac{\sqrt{6}}{3}$

En el cuadro sinóptico dado a continuación, resumimos los resultados anteriores.

CUADRO SINÓPTICO

Magnitud	Valor exacto	Valor decimal aproximado
<sup>216</sup> $l_6$	$\frac{2\sqrt{3}}{3} a$	1, 15 47 01... a
<sup>217</sup> $l_8$	$\sqrt{2} \cdot a$	1, 41 42 14... a
<sup>218</sup> $f$	$\frac{3\sqrt{2} - \sqrt{6}}{3} a$	0, 53 77 17... a
<sup>219</sup> $g$	$\frac{6 - 2\sqrt{3}}{3} a$	0, 84 52 99... a
<sup>220</sup> $h$	$\frac{3 - \sqrt{3}}{3} a$	0, 42 26 50... a
<sup>221</sup> $S$	$4 \cdot (6\sqrt{3} - 8) a^2$	9, 56 92 19... a <sup>2</sup>
<sup>222</sup> $V$	$\frac{8 \cdot (9 - 4\sqrt{3})}{9} a^3$	1, 84 15 97... a <sup>3</sup>
<sup>223</sup> $2\theta$	$\pi - \frac{1}{2} (2\varphi_8)$	125° 15' 51,7''
<sup>224</sup> $2\varphi_6$	$\frac{\pi}{2}$	90°
<sup>225</sup> $2\varphi_8$	$\text{arc sen } \frac{\sqrt{6}}{3}$	109° 28' 16,6''

The following table shows the results of the experiment conducted on the 10th of May 1900. The results are given in the form of a table, and the data is as follows:

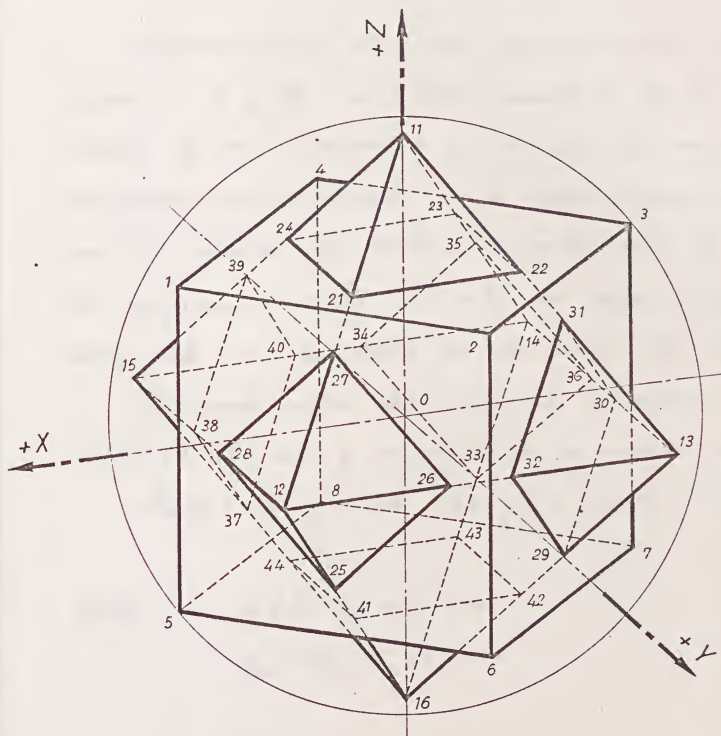
Time of day	Temperature of air	Temperature of water	Temperature of soil
7.00 AM	50° F	50° F	50° F
8.00 AM	55° F	55° F	55° F
9.00 AM	60° F	60° F	60° F
10.00 AM	65° F	65° F	65° F
11.00 AM	70° F	70° F	70° F
12.00 PM	75° F	75° F	75° F
1.00 PM	80° F	80° F	80° F
2.00 PM	85° F	85° F	85° F
3.00 PM	90° F	90° F	90° F
4.00 PM	95° F	95° F	95° F
5.00 PM	100° F	100° F	100° F
6.00 PM	105° F	105° F	105° F
7.00 PM	110° F	110° F	110° F
8.00 PM	115° F	115° F	115° F
9.00 PM	120° F	120° F	120° F
10.00 PM	125° F	125° F	125° F
11.00 PM	130° F	130° F	130° F
12.00 AM	135° F	135° F	135° F

FIGURA CORPÓREA

Para obtener esta figura se construye previamente el exaedro regular de 63,5 mm de lado, en cuyas caras se dibujará previamente la base de la pirámide complementaria correspondiente al octaedro; dicha base es un cuadrado de 32,9 mm de lado colocado en el interior de la cara del exaedro con su centro coincidente y sus vértices en las mediatrices de los lados de ésta.

Posteriormente se acoplarán a las caras del exaedro las seis pirámides, formadas cada una por cuatro triángulos equiláteros de 32,9 mm de lado.





*Poliedros regulares convexos conjugados*





FIGURE 1. A cube with internal lines and axes.

ENUNCIADO

Representar por el método gráfico-analítico, en los planos I, II y III, el poliedro resultante de la intersección de un dodecaedro y un icosaedro regulares recíprocamente conjugados y de esfera circunscrita común, siendo los vértices de cada uno de ellos las proyecciones desde el centro de dicha esfera, y sobre ésta, de los centros de las caras del otro.

Las coordenadas del centro de la esfera son :  
 $O(72, 72, 85)$  mm y el radio de la misma de 55 mm.

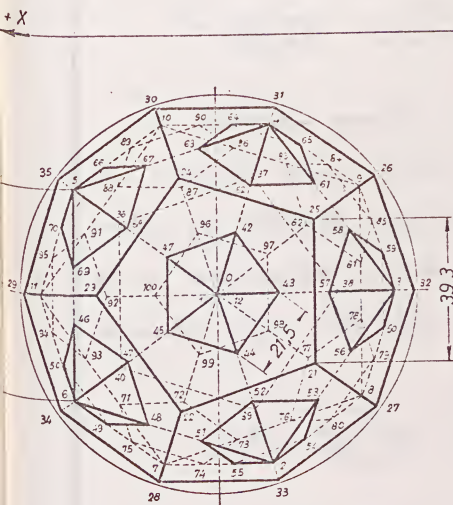
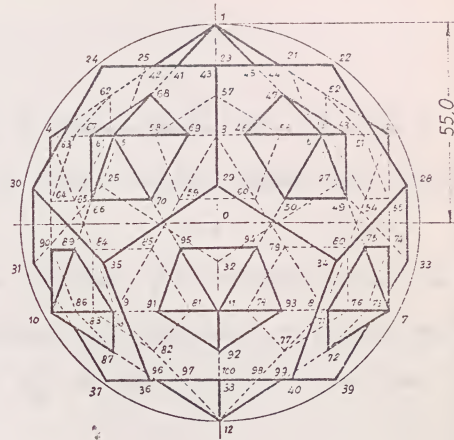
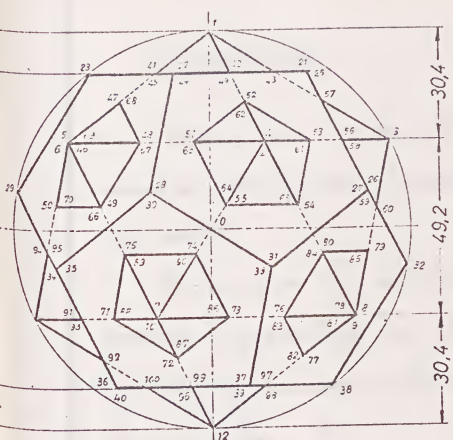
Dibujar en formato A3 y a escala 1:1.

DATOS

$O(72, 72, 85)$  mm

$a_{12} = a_{20} = 55$  mm.





## ENUNCIADO

Representar por el método gráfico-analítico en los planos I, II y III, el poliedro resultante de la intersección de un dodecaedro y un icosaedro regulares reciprocamente conjugados y de esfera circunscrita común, siendo los vértices de cada uno de ellos las proyecciones desde el centro de dicha esfera, y sobre ésta, de los centros de las caras del otro.

Las coordenadas del centro de la esfera son:  $O(72, 72, 85)$  mm y el radio de la misma de 55 mm.

Dibujar en formato A3v y a escala 1:1.

## NUMERACIÓN DE VÉRTICES

Icosaedro..... 1 al 12  
Dodecaedro..... 21 al 40  
Puntos de intersección..... 41 al 100

Propuesta	De entrega	Entregada	Calificación	(firma)	Escuela
Fecha:					Curso
Alumno					
Escala	Poliedros regulares convexos conjugados				Lámina 24
1:1					Curso 19 - 20



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PROCESO GRÁFICO

Procederemos previamente a la representación del dodecaedro e icosaedro conjugados, inscritos en una misma esfera de 55 mm de radio.

Comencemos por el icosaedro regular, que colocaremos en la posición representada en la lámina 5, siguiendo el proceso gráfico de la misma. El lado  $l_{20}$  de este icosaedro, será (ver lám. 5, fórm. 43)

$$l_{20} = \frac{a_{20}}{\frac{\sqrt{10 + 2\sqrt{5}}}{4}} = \frac{55}{0,951057} = 57,8 \text{ mm}$$

El dodecaedro regular, conjugado del anterior e inscrito en su misma esfera, tendrá una posición en relación con el icosaedro ya dibujado, simétrica con respecto a su eje paralelo a Z de la representada en la lámina 4. El lado  $l_{12}$  de este dodecaedro, será (ver lám. 4, fórm. 30)

$$l_{12} = \frac{a_{12}}{\frac{\sqrt{15} + \sqrt{3}}{4}} = \frac{55}{1,401259} = 39,3 \text{ mm.}$$

Plantado el problema de intersección de ambos poliedros, se puede obtener el resultado por aplicación del método de intersección de caras o de aristas cuyo fundamento se expone en la Geometría Descriptiva.

Esta intersección se reduce a la penetración de los ángulos

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sólidos del icosaedro en las caras del dodecaedro que, por razones de simetría en la posición relativa de ambos poliedros, son todas iguales y en forma de pentágonos regulares. Combinando las proyecciones de los dos poliedros en los planos I y II, pueden obtenerse las proyecciones de estos pentágonos regulares situados en cada cara del dodecaedro (líneas de penetración del icosaedro en el dodecaedro) unas directamente y las restantes mediante giros adecuados de una de las aristas de dicho ángulo sólido. De las proyecciones en I y II se obtiene fácilmente la proyección sobre III.

El sólido resultante se compone básicamente de un dodecaedro regular en cuyas caras, y centradas con ellas, se apoyan seis pirámides rectas de base pentagonal regular, y caras laterales en forma de triángulos equiláteros. Estas pirámides son todas iguales y en base sobre cada cara del dodecaedro tiene el centro coincidente con el de dicha cara; los vértices del pentágono base de la pirámide están sobre las mediatrices de los lados de la cara correspondiente del dodecaedro.

### PROCESO GRÁFICO-ANALÍTICO

Para facilitar el dibujo y conseguir una mayor exactitud en su trazado, calcularemos las siguientes magnitudes en función del radio  $a$  de la esfera circunscrita común:

The first session of the General Assembly of the  
Presbyterian Church of the United States was held  
at the city of New York, on the 1st day of  
September, 1852. The assembly was composed  
of the following members: [illegible]  
The assembly was organized by the reading of  
the Declaration of Sentiments, and the adoption  
of the following resolutions: [illegible]  
The assembly then proceeded to the election of  
officers, and the following were elected: [illegible]

The assembly then proceeded to the consideration  
of the report of the Committee on the  
[illegible] and the following resolutions were  
adopted: [illegible]  
The assembly then proceeded to the consideration  
of the report of the Committee on the  
[illegible] and the following resolutions were  
adopted: [illegible]  
The assembly then proceeded to the consideration  
of the report of the Committee on the  
[illegible] and the following resolutions were  
adopted: [illegible]  
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adopted: [illegible]  
The assembly then proceeded to the consideration  
of the report of the Committee on the  
[illegible] and the following resolutions were  
adopted: [illegible]  
The assembly then proceeded to the consideration  
of the report of the Committee on the  
[illegible] and the following resolutions were  
adopted: [illegible]

$l_{12}$  = Lado del dodecaedro

$l_{20}$  = Lado del icosaedro

$f$  = Lado del pentágono regular intersección de un ángulo sólido del icosaedro con una cara del dodecaedro.

$i$  = Radio de la circunferencia circunscrita al pentágono anterior.

$h$  = Altura de la pirámide exterior a una cara del dodecaedro

$S$  = Superficie lateral

$V$  = Volumen

$2\theta$  = Ángulo rectilíneo del diedro formado en la intersección de una cara del dodecaedro con la correspondiente del icosaedro.

Lado " $l_{12}$ " del dodecaedro

Ya lo hemos determinado anteriormente (se deduce de la fórmula 30, lám. 4)

$$l_{12} = \frac{a}{\frac{\sqrt{15} + \sqrt{3}}{4}} = \frac{\sqrt{15} - \sqrt{3}}{3} a$$

Desarrollo del cálculo anterior:  $l_{12} = \frac{a}{\frac{\sqrt{15} + \sqrt{3}}{4}} = \frac{4}{\sqrt{15} + \sqrt{3}} a =$

$$= \frac{4(\sqrt{15} - \sqrt{3})}{15 - 3} a = \boxed{\frac{\sqrt{15} - \sqrt{3}}{3} a}$$





Lado " $l_{20}$ " del icosaedro

Iguualmente hemos visto, que (lámin. 5, fórmula 43)

$$l_{20} = \frac{a}{\frac{\sqrt{10+2\sqrt{5}}}{4}} = \frac{\sqrt{10 \times (5-\sqrt{5})}}{5} a$$

$$\begin{aligned} \text{Desarrollo del cálculo anterior: } l_{20} &= \frac{a}{\frac{\sqrt{10+2\sqrt{5}}}{4}} = \frac{4}{\sqrt{10+2\sqrt{5}}} a = \\ &= \frac{4 \sqrt{10+2\sqrt{5}}}{10+2\sqrt{5}} a = \frac{4 \sqrt{10+2\sqrt{5}} \times (10-2\sqrt{5})}{100-20} a = \frac{4 \sqrt{(10+2\sqrt{5})(10-2\sqrt{5})^2}}{80} a = \\ &= \frac{\sqrt{(100-20)(10-2\sqrt{5})}}{20} a = \frac{\sqrt{80 \times 2 \times (5-\sqrt{5})}}{20} a = \frac{4 \sqrt{10(5-\sqrt{5})}}{20} a = \boxed{\frac{\sqrt{10 \times (5-\sqrt{5})}}{5} a} \end{aligned}$$

Altura " $h$ " de la pirámide exterior a una cara del dodecaedro.

En la lámina 24 se puede observar que dicha altura es la diferencia entre el radio  $a$  de la esfera circunscrita y el radio  $c_{12}$  de la esfera inscrita en el dodecaedro (ver lámin. 4, fórm. 32); su valor será pues

$$h = a - \sqrt{\frac{11\sqrt{5}+25}{40}} l_{12} = a - \sqrt{\frac{11\sqrt{5}+25}{40}} \times \left( \frac{\sqrt{15}-\sqrt{3}}{3} \right) a = \left( 1 - \sqrt{\frac{5+2\sqrt{5}}{15}} \right) a$$

$$\text{Desarrollo del cálculo anterior: } h = a - \sqrt{\frac{11\sqrt{5}+25}{40}} \times \frac{\sqrt{15}-\sqrt{3}}{3} a =$$

$$= \left( 1 - \sqrt{\frac{11\sqrt{5}+25}{40} \times \left( \frac{\sqrt{15}-\sqrt{3}}{3} \right)^2} \right) a = \left( 1 - \sqrt{\frac{11\sqrt{5}+25}{40} \times \frac{15+3-2\sqrt{45}}{9}} \right) a =$$

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the thirteenth of these is the fact that the

the fourteenth of these is the fact that the

$$= \left( 1 - \sqrt{\frac{11\sqrt{5} + 25}{40} \times \frac{18 - 6\sqrt{5}}{9}} \right) a = \left( 1 - \sqrt{\frac{(11\sqrt{5} + 25)(3 - \sqrt{5})}{60}} \right) a =$$

$$= \left( 1 - \sqrt{\frac{33\sqrt{5} + 75 - 55 - 25\sqrt{5}}{60}} \right) a = \left( 1 - \sqrt{\frac{20 + 8\sqrt{5}}{60}} \right) a = \boxed{\left( 1 - \sqrt{\frac{5 + 2\sqrt{5}}{15}} \right) a}$$

Lado "f" del pentágono regular intersección de un ángulo sólido del icosaedro, con una cara del dodecaedro.

En la lámina 24 podemos comprobar que las cinco caras concurrentes en un vértice del icosaedro forman una pirámide recta, regular, pentagonal de altura "g<sub>20</sub>", cuyo valor es (ver lám. 5, fórmula 20)

$$g_{20} = \sqrt{\frac{5 - \sqrt{5}}{10}} l_{20}$$

Esta pirámide es cortada por la correspondiente cara del dodecaedro, cuyo plano es paralelo a la base de aquella, produciendo otra pirámide semejante a la primera. De la semejanza de ambas pirámides se deduce que

$$\frac{l_{20}}{g_{20}} = \frac{f}{h} \quad \text{de donde} \quad f = \frac{h \times l_{20}}{g_{20}}$$

El valor de "h" deducido anteriormente, y el de g<sub>20</sub> (ver lám. 5, fórm. 20), son los siguientes:

$$h = \left( 1 - \sqrt{\frac{5 + 2\sqrt{5}}{15}} \right) a \quad \text{y} \quad g_{20} = \sqrt{\frac{5 - \sqrt{5}}{10}} l_{20}$$

Let  $x = \sqrt{a}$  and  $y = \sqrt{b}$  then  $x^2 = a$  and  $y^2 = b$

$$\frac{1}{x} = \frac{1}{\sqrt{a}} = \frac{\sqrt{a}}{a} \quad \frac{1}{y} = \frac{1}{\sqrt{b}} = \frac{\sqrt{b}}{b}$$

Adding these two equations we get  $\frac{1}{x} + \frac{1}{y} = \frac{\sqrt{a}}{a} + \frac{\sqrt{b}}{b}$

Now we can rationalize the denominator by multiplying both sides by  $\frac{a}{a}$  and  $\frac{b}{b}$

$$\frac{1}{x} + \frac{1}{y} = \frac{\sqrt{a}}{a} + \frac{\sqrt{b}}{b} \Rightarrow \frac{1}{x} + \frac{1}{y} = \frac{\sqrt{a}b + \sqrt{b}a}{ab}$$

$$\frac{1}{x} + \frac{1}{y} = \frac{\sqrt{a}b + \sqrt{b}a}{ab}$$

Now we can simplify the expression by factoring out  $\sqrt{ab}$  from the numerator

$$\frac{1}{x} + \frac{1}{y} = \frac{\sqrt{ab}(\sqrt{b} + \sqrt{a})}{ab}$$

$$\frac{1}{x} + \frac{1}{y} = \frac{\sqrt{ab}(\sqrt{a} + \sqrt{b})}{ab}$$

Now we can simplify the expression by dividing both sides by  $\sqrt{ab}$

$$\frac{1}{x} + \frac{1}{y} = \frac{\sqrt{a} + \sqrt{b}}{\sqrt{ab}}$$



que sustituido en la anterior, nos da

$$f = \frac{h \cdot l_{20}}{g_{20}} = \frac{\left(1 - \frac{\sqrt{5+2\sqrt{5}}}{15}\right) a l_{20}}{\frac{\sqrt{5-\sqrt{5}}}{10} l_{20}} = \frac{1 - \frac{\sqrt{5+2\sqrt{5}}}{15}}{\frac{\sqrt{5-\sqrt{5}}}{10}} a = \frac{3\sqrt{10+2\sqrt{5}} - (3\sqrt{3} + \sqrt{15})}{6} a$$

Desarrollo del cálculo anterior:  $f = \frac{1 - \frac{\sqrt{5+2\sqrt{5}}}{15}}{\frac{\sqrt{5-\sqrt{5}}}{10}} a =$

$$\begin{aligned} & \frac{\left(1 - \frac{\sqrt{5+2\sqrt{5}}}{15}\right) \sqrt{\frac{5-\sqrt{5}}{10}}}{\frac{5-\sqrt{5}}{10}} a = \frac{10 \left(1 - \frac{\sqrt{5+2\sqrt{5}}}{15}\right) \sqrt{\frac{5-\sqrt{5}}{10}} (5+\sqrt{5})}{20} a = \\ & = \frac{\left(1 - \frac{\sqrt{5+2\sqrt{5}}}{15}\right) \sqrt{\frac{5-\sqrt{5}}{10}} (5+\sqrt{5})^2}{2} a = \frac{\left(1 - \frac{\sqrt{5+2\sqrt{5}}}{15}\right) \sqrt{\frac{20(5+\sqrt{5})}{10}}}{2} a = \\ & = \left(1 - \frac{\sqrt{5+2\sqrt{5}}}{15}\right) \sqrt{\frac{2(5+\sqrt{5})}{4}} a = \left(1 - \frac{\sqrt{5+2\sqrt{5}}}{15}\right) \sqrt{\frac{5+\sqrt{5}}{2}} a = \\ & = \left[ \sqrt{\frac{5+\sqrt{5}}{2}} - \sqrt{\frac{(5+2\sqrt{5})(5+\sqrt{5})}{30}} \right] a = \left[ \sqrt{\frac{5+\sqrt{5}}{2}} - \sqrt{\frac{25+10\sqrt{5}+5\sqrt{5}+10}{30}} \right] a = \\ & = \left[ \sqrt{\frac{5+\sqrt{5}}{2}} - \sqrt{\frac{35+15\sqrt{5}}{30}} \right] a = \left[ \sqrt{\frac{5+\sqrt{5}}{2}} - \sqrt{\frac{7+3\sqrt{5}}{6}} \right] a = \left[ \sqrt{\frac{5+\sqrt{5}}{2}} - \frac{\sqrt{7+3\sqrt{5}}}{\sqrt{6}} \right] a = \\ & = \left( \sqrt{\frac{5+\sqrt{5}}{2}} - \frac{\sqrt{\frac{9}{2}} + \sqrt{\frac{5}{2}}}{\sqrt{6}} \right) a = \left( \sqrt{\frac{5+\sqrt{5}}{2}} - \left( \sqrt{\frac{9}{12}} + \sqrt{\frac{5}{12}} \right) \right) a = \\ & = \left[ \sqrt{\frac{5+\sqrt{5}}{2}} - \left( \frac{3}{2\sqrt{3}} + \frac{\sqrt{5}}{2\sqrt{3}} \right) \right] a = \left[ \sqrt{\frac{5+\sqrt{5}}{2}} - \frac{3+\sqrt{5}}{2\sqrt{3}} \right] a = \left[ \sqrt{\frac{5+\sqrt{5}}{2}} - \frac{3\sqrt{3}+\sqrt{15}}{6} \right] a = \\ & = \left( \frac{\sqrt{5+\sqrt{5}}}{\sqrt{2}} - \frac{3\sqrt{3}+\sqrt{15}}{6} \right) a = \left( \frac{\sqrt{2}\sqrt{5+\sqrt{5}}}{2} - \frac{3\sqrt{3}+\sqrt{15}}{6} \right) a = \left( \frac{\sqrt{10+2\sqrt{5}}}{2} - \frac{3\sqrt{3}+\sqrt{15}}{6} \right) a = \\ & = \frac{3\sqrt{10+2\sqrt{5}} - (3\sqrt{3} + \sqrt{15})}{6} a \end{aligned}$$

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Radio "i" de la circunferencia circunscrita al pentágono base

En Geometría se obtiene el radio  $r$  de la circunferencia circunscrita a un pentágono regular, por la expresión

$$r = \frac{\sqrt{5 + \sqrt{5}}}{10} l_5$$

que aplicada al caso que nos ocupa, nos da

$$i = \frac{\sqrt{5 + \sqrt{5}}}{10} \times \frac{3\sqrt{10 + 2\sqrt{5}} - (3\sqrt{3} + \sqrt{15})}{6} a = \left( \frac{\sqrt{5} + 1}{2} - \sqrt{\frac{25 + 11\sqrt{5}}{30}} \right) a$$

Desarrollo del cálculo anterior:

$$\begin{aligned} i &= \frac{\sqrt{5 + \sqrt{5}}}{10} \times \frac{3\sqrt{10 + 2\sqrt{5}} - (3\sqrt{3} + \sqrt{15})}{6} a = \left[ \frac{\sqrt{5 + \sqrt{5}}}{10} \times \frac{\sqrt{2(5 + \sqrt{5})}}{2} - \right. \\ &\quad \left. - \frac{\sqrt{5 + \sqrt{5}}}{10} \times \frac{3\sqrt{3} + \sqrt{3}\sqrt{5}}{6} \right] a = \left[ \frac{\sqrt{5 + \sqrt{5}}}{10} \times \sqrt{\frac{5 + \sqrt{5}}{2}} - \frac{\sqrt{5 + \sqrt{5}}}{10} \times \frac{\sqrt{3}(3 + \sqrt{5})}{6} \right] a = \\ &= \left[ \frac{\sqrt{(5 + \sqrt{5})^2}}{20} - \frac{\sqrt{3}}{6} \sqrt{\frac{(5 + \sqrt{5})(3 + \sqrt{5})^2}{10}} \right] a = \frac{5 + \sqrt{5}}{2\sqrt{5}} - \frac{\sqrt{3}}{6} \sqrt{\frac{(5 + \sqrt{5})(9 + 5 + 6\sqrt{5})}{10}} a = \\ &= \left[ \frac{\sqrt{5}(5 + \sqrt{5})}{10} - \frac{\sqrt{3}}{6} \sqrt{\frac{2(5 + \sqrt{5})(7 + 3\sqrt{5})}{10}} \right] a = \left[ \frac{5\sqrt{5} + 5}{10} - \frac{\sqrt{3}}{6} \sqrt{\frac{35 + 7\sqrt{5} + 15\sqrt{5} + 15}{5}} \right] a = \\ &= \left[ \frac{\sqrt{5} + 1}{2} - \frac{\sqrt{3}}{6} \sqrt{\frac{50 + 22\sqrt{5}}{5}} \right] a = \left[ \frac{\sqrt{5} + 1}{2} - \sqrt{\frac{3 \times 2 \times (25 + 11\sqrt{5})}{6^2 \times 5}} \right] a = \\ &= \left[ \frac{\sqrt{5} + 1}{2} - \sqrt{\frac{25 + 11\sqrt{5}}{30}} \right] a \end{aligned}$$

The following is a list of the names of the persons who have been  
 admitted to the office of the Secretary of the Board of Education  
 since the last meeting of the Board.

(List of names)

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### Superficie lateral $S$

A cada cara del dodecaedro, habrá que deducir el área del pentágono base de la pirámide que se apoya en ella: En geometría se obtiene el área de un pentágono regular, en función de su lado, por la fórmula

$$S_5 = \frac{\sqrt{25 + 10\sqrt{5}}}{4} l_5^2$$



que aplicada al caso que nos ocupa, nos permite obtener el área lateral  $S_1$  de las caras del dodecaedro, como sigue:

$$S_1 = 12 \times \left[ \frac{\sqrt{25 + 10\sqrt{5}}}{4} \times \left( \frac{\sqrt{15} - \sqrt{3}}{3} a \right)^2 - \frac{\sqrt{25 + 10\sqrt{5}}}{4} \times \left( \frac{3\sqrt{10 + 2\sqrt{5}} - (3\sqrt{3} + \sqrt{15})}{6} a \right)^2 \right] =$$

$$= 5 \times \left[ 7\sqrt{3} + 3\sqrt{15} - \sqrt{10(25 + 11\sqrt{5})} \right] a^2$$

Desarrollo del cálculo anterior:

$$S_1 = 12 \times \left[ \frac{\sqrt{25 + 10\sqrt{5}}}{4} \times \left( \frac{\sqrt{15} - \sqrt{3}}{3} a \right)^2 - \frac{\sqrt{25 + 10\sqrt{5}}}{4} \times \left( \frac{3\sqrt{10 + 2\sqrt{5}} - (3\sqrt{3} + \sqrt{15})}{6} a \right)^2 \right] =$$

$$= 12 \times \frac{\sqrt{25 + 10\sqrt{5}}}{4} a^2 \left[ \left( \frac{\sqrt{15} - \sqrt{3}}{3} \right)^2 - \left( \frac{3\sqrt{10 + 2\sqrt{5}} - (3\sqrt{3} + \sqrt{15})}{6} \right)^2 \right] =$$

$$= 3\sqrt{25 + 10\sqrt{5}} a^2 \left( \frac{15 + 3 - 2\sqrt{45}}{9} - \frac{9(10 + 2\sqrt{5}) + (3\sqrt{3} + \sqrt{15})^2 - 6\sqrt{10 + 2\sqrt{5}} \times (3\sqrt{3} + \sqrt{15})}{36} \right) =$$



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The University of Chicago is a private research university in Chicago, Illinois. It was founded in 1837 and is one of the oldest and most prominent universities in the United States. The university is known for its commitment to academic excellence and its diverse student body.



$$\frac{1}{2} \left( \frac{1}{2} + \frac{1}{2} \right) = 1$$

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$$\frac{1}{2} \left( \frac{1}{2} + \frac{1}{2} \right) = 1$$

$$\begin{aligned}
&= 3\sqrt{25+10\sqrt{5}} a^2 \left( \frac{18-6\sqrt{5}}{9} - \frac{90+18\sqrt{5}+(27+15+6\sqrt{45})-6\sqrt{2(5+\sqrt{5})(3\sqrt{3}+\sqrt{15})^2}}{36} \right) = \\
&= 3\sqrt{25+10\sqrt{5}} a^2 \left( \frac{6-2\sqrt{5}}{3} - \frac{90+18\sqrt{5}+42+18\sqrt{5}-6\sqrt{2(5+\sqrt{5})(27+15+6\sqrt{45})}}{36} \right) = \\
&= 3\sqrt{25+10\sqrt{5}} a^2 \left( \frac{6-2\sqrt{5}}{3} - \frac{132+36\sqrt{5}-6\sqrt{2(5+\sqrt{5})(42+18\sqrt{5})}}{36} \right) = \\
&= 3\sqrt{25+10\sqrt{5}} a^2 \left( \frac{6-2\sqrt{5}}{3} - \frac{22+6\sqrt{5}-\sqrt{2 \times 6 \times (5+\sqrt{5})(7+3\sqrt{5})}}{6} \right) = \\
&= 3\sqrt{25+10\sqrt{5}} a^2 \left( \frac{6-2\sqrt{5}}{3} - \frac{22+6\sqrt{5}-2\sqrt{3(35+7\sqrt{5}+15\sqrt{5}+15)}}{6} \right) = \\
&= 3\sqrt{25+10\sqrt{5}} a^2 \left( \frac{6-2\sqrt{5}}{3} - \frac{11+3\sqrt{5}-\sqrt{3(50+22\sqrt{5})}}{3} \right) = \\
&= \sqrt{25+10\sqrt{5}} a^2 \left( 6-2\sqrt{5} - (11+3\sqrt{5}-\sqrt{6(25+11\sqrt{5})}) \right) = \\
&= \sqrt{25+10\sqrt{5}} a^2 \left( 6-2\sqrt{5}-11-3\sqrt{5}+\sqrt{6(25+11\sqrt{5})} \right) = \\
&= \sqrt{25+10\sqrt{5}} a^2 \left( \sqrt{6(25+11\sqrt{5})} - 5(1+\sqrt{5}) \right) = \\
&= \left[ \sqrt{25+10\sqrt{5}} \times \sqrt{6(25+11\sqrt{5})} - \sqrt{25+10\sqrt{5}} \times 5(1+\sqrt{5}) \right] a^2 = \\
&= \left[ \sqrt{(25+10\sqrt{5})(25+11\sqrt{5}) \times 6} - 5\sqrt{(25+10\sqrt{5})(1+\sqrt{5})^2} \right] a^2 = \\
&= \left[ \sqrt{6(25^2+11 \times 25\sqrt{5}+250\sqrt{5}+110 \times 5)} - 5\sqrt{(25+10\sqrt{5})(1+5+2\sqrt{5})} \right] a^2 = \\
&= \left[ \sqrt{6 \times 25(25+11\sqrt{5}+10\sqrt{5}+22)} - 5\sqrt{2(25+10\sqrt{5})(3+\sqrt{5})} \right] a^2 =
\end{aligned}$$

1. Introduction

The purpose of this study is to investigate the effects of the independent variable on the dependent variable.

The study was conducted in a laboratory setting with a sample size of 30 participants.

The results of the study indicate that there is a significant positive correlation between the variables.

The data was analyzed using statistical methods, including t-tests and ANOVA.

The findings suggest that the independent variable has a significant impact on the dependent variable.

These results are consistent with previous research in this field.

The study has several limitations, including a small sample size and a lack of control over certain variables.

Future research should aim to address these limitations and further explore the relationship between the variables.

The study was funded by the National Science Foundation.

The authors would like to thank the participants for their contribution to the study.

The data for this study is available upon request.

The study was approved by the Institutional Review Board.

The authors have no conflicts of interest.

$$\begin{aligned}
 &= \left[ 5 \sqrt{6(47 + 21\sqrt{5})} - 5 \sqrt{2(75 + 30\sqrt{5} + 25\sqrt{5} + 50)} \right] a^2 = \\
 &= \left[ 5 \sqrt{6} \sqrt{47 + 21\sqrt{5}} - 5 \sqrt{2(125 + 55\sqrt{5})} \right] a^2 = \quad \text{y siendo } 47^2 - (21\sqrt{5})^2 = 2^2 \\
 &= \left[ 5 \sqrt{6} \left( \sqrt{\frac{49}{2}} + \sqrt{\frac{45}{2}} \right) - 5 \sqrt{10(25 + 11\sqrt{5})} \right] a^2 = \\
 &= \left[ 5 \left( \frac{\sqrt{6} \sqrt{49}}{\sqrt{2}} + \frac{\sqrt{6} \sqrt{45}}{\sqrt{2}} \right) - 5 \sqrt{10(25 + 11\sqrt{5})} \right] a^2 = \\
 &= 5 \left[ 7\sqrt{3} + \sqrt{3} \times 3\sqrt{5} - \sqrt{10(25 + 11\sqrt{5})} \right] a^2 = \\
 &= 5 \left[ 7\sqrt{3} + 3\sqrt{15} - \sqrt{10(25 + 11\sqrt{5})} \right] a^2
 \end{aligned}$$

Al área anteriormente calculada, habrá que añadirle el área  $S_2$  de las doce pirámides rectas pentagonales, cuyas caras laterales son triángulos equiláteros de lado "f". La superficie será,

$$\begin{aligned}
 S_2 &= 12 \times \frac{5}{2} f \times \frac{\sqrt{3}}{2} f = 15\sqrt{3} f^2 = 15\sqrt{3} \times \left( \frac{3\sqrt{10+2\sqrt{5}} - (3\sqrt{3} + \sqrt{15})}{6} a \right)^2 = \\
 &= 5 (11\sqrt{3} + 3\sqrt{15} - 3\sqrt{2(25 + 11\sqrt{5})}) a^2
 \end{aligned}$$

Desarrollo del cálculo anterior:

$$S_2 = 15\sqrt{3} \left( \frac{3\sqrt{10+2\sqrt{5}} - (3\sqrt{3} + \sqrt{15})}{6} a \right)^2 =$$

Question 1: Solve the system of linear equations.

$$\begin{cases} x + y = 10 \\ 2x - y = 4 \end{cases}$$

$$\Rightarrow \begin{pmatrix} 1 & 1 \\ 2 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 10 \\ 4 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} 1 & 1 \\ 2 & -1 \end{pmatrix} \xrightarrow{R_2 - 2R_1} \begin{pmatrix} 1 & 1 \\ 0 & -3 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 10 \\ -16 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} 1 & 1 \\ 0 & -3 \end{pmatrix} \xrightarrow{R_1 + R_2} \begin{pmatrix} 1 & 0 \\ 0 & -3 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -6 \\ -16 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -6 \\ \frac{16}{3} \end{pmatrix}$$

Therefore, the solution to the system of linear equations is  $x = -6$  and  $y = \frac{16}{3}$ .  
 The solution set is  $\left\{ \begin{pmatrix} -6 \\ \frac{16}{3} \end{pmatrix} \right\}$ .

Question 2: Find the inverse of the matrix  $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ .

$$A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$$

$$\Rightarrow |A| = \begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} = 4 - 6 = -2$$

$$\Rightarrow A^{-1} = \frac{1}{|A|} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix} = \frac{1}{-2} \begin{pmatrix} 4 & -2 \\ -3 & 1 \end{pmatrix}$$



$$\begin{aligned}
 &= 15 \sqrt{3} a^2 \left( \frac{9(10+2\sqrt{5}) + (3\sqrt{3} + \sqrt{15})^2 - 6\sqrt{10+2\sqrt{5}} \cdot (3\sqrt{3} + \sqrt{15})}{36} \right) = \\
 &= 15 \sqrt{3} a^2 \left( \frac{18(5+\sqrt{5}) + (27+15+6\sqrt{45}) - 6\sqrt{2(5+\sqrt{5})(3\sqrt{3} + \sqrt{15})^2}}{36} \right) = \\
 &= 15 \sqrt{3} a^2 \left( \frac{18(5+\sqrt{5}) + 6(7+3\sqrt{5}) - 6\sqrt{2(5+\sqrt{5})(27+15+6\sqrt{45})}}{36} \right) = \\
 &= 15 \sqrt{3} a^2 \left( \frac{3(5+\sqrt{5}) + (7+3\sqrt{5}) - \sqrt{2(5+\sqrt{5})(42+18\sqrt{5})}}{6} \right) = \\
 &= 15 \sqrt{3} a^2 \left( \frac{15+3\sqrt{5} + 7+3\sqrt{5} - \sqrt{2(5+\sqrt{5})(7+3\sqrt{5}) \cdot 6}}{6} \right) = \\
 &= 15 \sqrt{3} a^2 \left( \frac{22+6\sqrt{5} - 2\sqrt{3(35+7\sqrt{5}+15\sqrt{5}+15)}}{6} \right) = \\
 &= 15 \sqrt{3} a^2 \left( \frac{11+3\sqrt{5} - \sqrt{3(50+22\sqrt{5})}}{3} \right) = 5 \sqrt{3} (11+3\sqrt{5} - \sqrt{3 \cdot 2(25+11\sqrt{5})}) a^2 = \\
 &= 5 (11\sqrt{3} + 3\sqrt{15} - \sqrt{6(25+11\sqrt{5})}) a^2 = \boxed{5 (11\sqrt{3} + 3\sqrt{15} - 3\sqrt{2(25+11\sqrt{5})}) a^2}
 \end{aligned}$$

Al área total de poliedro, valdrá pues

$$\begin{aligned}
 S &= S_1 + S_2 = 5(7\sqrt{3} + 3\sqrt{15} - \sqrt{10(25+11\sqrt{5})}) a^2 + 5(11\sqrt{3} + 3\sqrt{15} - 3\sqrt{2(25+11\sqrt{5})}) a^2 = \\
 &= 5(18\sqrt{3} + 6\sqrt{15} - \sqrt{10(25+11\sqrt{5})} - 3\sqrt{2(25+11\sqrt{5})}) a^2
 \end{aligned}$$

### Volumen V

El volumen V de este poliedro se puede determinar, sumando al volumen  $V_1$  del dodecaedro regular, el volumen  $V_2$  de

1. The first part of the paper is devoted to a general discussion of the problem. It is shown that the problem is of great importance in the theory of the structure of the atom. The problem is to find the wave function of the electron in the atom. This is done by solving the Schrödinger equation. The solution is found by the method of separation of variables. The wave function is then used to find the probability of finding the electron in a certain region of space. This is done by integrating the square of the wave function over the region. The result is then compared with the experimental results. It is found that the theory is in good agreement with the experiment. This shows that the theory is correct. The second part of the paper is devoted to a detailed discussion of the problem. It is shown that the problem is of great importance in the theory of the structure of the atom. The problem is to find the wave function of the electron in the atom. This is done by solving the Schrödinger equation. The solution is found by the method of separation of variables. The wave function is then used to find the probability of finding the electron in a certain region of space. This is done by integrating the square of the wave function over the region. The result is then compared with the experimental results. It is found that the theory is in good agreement with the experiment. This shows that the theory is correct.

Las doce pirámides que se apoyan sobre sus caras.

Así pues, tendremos que (ver lám. 4, fórm. 11)

$$V_1 = \frac{7\sqrt{5}+15}{4} \left(\frac{1}{2}\right)^3 = \frac{7\sqrt{5}+15}{4} \times \left(\frac{\sqrt{15}-\sqrt{3}}{3} a\right)^3 = \frac{10\sqrt{3}+2\sqrt{15}}{9} a^3$$

Desarrollo del cálculo anterior:  $V_1 = \frac{7\sqrt{5}+15}{4} \times \left(\frac{\sqrt{15}-\sqrt{3}}{3} a\right)^3 =$

$$= \frac{7\sqrt{5}+15}{4} \times \frac{15\sqrt{15} - 3 \times 15 \times \sqrt{3} + 3\sqrt{15} \times 3 - 3\sqrt{3}}{3^3} a^3 = \frac{7\sqrt{5}+15}{4} \times$$

$$\times \frac{5\sqrt{15} - 15\sqrt{3} + 3\sqrt{15} - \sqrt{3}}{3^2} a^3 = \frac{7\sqrt{5}+15}{4} \times \frac{8\sqrt{15} - 16\sqrt{3}}{9} a^3 =$$

$$= \frac{8 \times (7\sqrt{5}+15)(\sqrt{15}-2\sqrt{3})}{4 \times 9} a^3 = \frac{2 \times (7\sqrt{15 \times 5} + 15\sqrt{15} - 14\sqrt{5 \times 3} - 30\sqrt{3})}{9} a^3 =$$

$$= \frac{2 \times (7 \times 5 \times \sqrt{3} + 15\sqrt{15} - 14\sqrt{15} - 30\sqrt{3})}{9} a^3 = \frac{2(5\sqrt{3} + \sqrt{15})}{9} a^3 =$$

$$= \boxed{\frac{10\sqrt{3}+2\sqrt{15}}{9} a^3}$$

El volumen  $V_2$  de las doce pirámides pentagonales, será:

$$V_2 = 12 \left( S_5 \times \frac{h}{3} \right) = 12 \times \frac{\sqrt{25+10\sqrt{5}}}{4} \times \left( \frac{3\sqrt{10+2\sqrt{5}} - (3\sqrt{3}+\sqrt{15})}{6} a \right)^2 \times$$

$$\times \frac{1}{3} \left( 1 - \sqrt{\frac{5+2\sqrt{5}}{15}} \right) a = \left[ \frac{\sqrt{10(745+331\sqrt{5})}}{3} + \frac{\sqrt{10(445+199\sqrt{5})}}{3} - \frac{190\sqrt{3}+82\sqrt{15}}{9} \right] a^3$$

Desarrollo del cálculo anterior:

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$$\begin{aligned}
 V_2 &= 12 \times \frac{\sqrt{25+10\sqrt{5}}}{4} \times \left( \frac{3\sqrt{10+2\sqrt{5}} - (3\sqrt{3} + \sqrt{15})}{6} a \right)^2 \times \frac{\left(1 - \sqrt{\frac{5+2\sqrt{5}}{15}}\right)}{3} a = \\
 &= \frac{12}{4 \times 6^2 \times 3} \times \sqrt{5(5+2\sqrt{5})} \times \left(1 - \sqrt{\frac{5+2\sqrt{5}}{15}}\right) \times \left(3\sqrt{10+2\sqrt{5}} - (3\sqrt{3} + \sqrt{15})\right)^2 a^3 = \\
 &= \frac{1}{36} \times \left[ \sqrt{5(5+2\sqrt{5})} - \frac{\sqrt{5} \sqrt{5+2\sqrt{5}} \times \sqrt{5+2\sqrt{5}}}{\sqrt{15}} \right] \times \left[ 9 \times (10+2\sqrt{5}) + (3\sqrt{3} + \sqrt{15})^2 - \right. \\
 &\quad \left. - 6 \sqrt{2(5+\sqrt{5})} \times (3\sqrt{3} + \sqrt{15}) \right] a^3 = \frac{1}{36} \left[ \sqrt{5(5+2\sqrt{5})} - \frac{(5+2\sqrt{5}) \cdot \sqrt{5}}{\sqrt{15}} \right] \times \\
 &\quad \times \left[ 18(5+\sqrt{5}) + (27+15+6\sqrt{45}) - 6 \sqrt{2(5+\sqrt{5})} (3\sqrt{3} + \sqrt{15})^2 \right] a^3 = \\
 &= \frac{1}{36} \left[ \sqrt{5(5+2\sqrt{5})} - \frac{5+2\sqrt{5}}{\sqrt{3}} \right] \left[ 90+18\sqrt{5}+42+18\sqrt{5}-6\sqrt{2(5+\sqrt{5})(27+15+6\sqrt{45})} \right] a^3 = \\
 &= \frac{1}{36} \left[ \frac{\sqrt{15(5+2\sqrt{5})} - (5+2\sqrt{5})}{\sqrt{3}} \right] \times \left[ 132+36\sqrt{5}-6\sqrt{2(5+\sqrt{5}) \times 6(7+3\sqrt{5})} \right] a^3 = \\
 &= \frac{1}{36} \left[ \frac{\sqrt{15(5+2\sqrt{5})} - (5+2\sqrt{5})}{\sqrt{3}} \right] \times \left[ 132+36\sqrt{5}-12\sqrt{3(5+\sqrt{5})(7+3\sqrt{5})} \right] a^3 = \\
 &= \frac{1}{3} \left[ \frac{\sqrt{15(5+2\sqrt{5})} - (5+2\sqrt{5})}{\sqrt{3}} \right] \times \left[ 11+3\sqrt{5}-\sqrt{3(35+7\sqrt{5}+15\sqrt{5}+15)} \right] a^3 = \\
 &= \frac{1}{3\sqrt{3}} \left( (\sqrt{15(5+2\sqrt{5})} - (5+2\sqrt{5})) \times [11+3\sqrt{5}-\sqrt{3(50+22\sqrt{5})}] \right) a^3 =
 \end{aligned}$$





$$= \frac{\sqrt{3}}{9} \left[ \sqrt{15(5+2\sqrt{5})} - (5+2\sqrt{5}) \right] \times \left[ (11+3\sqrt{5}) - \sqrt{6(25+11\sqrt{5})} \right] a^3 =$$

$$= \frac{\sqrt{3}}{9} \left( (11+3\sqrt{5}) \sqrt{15(5+2\sqrt{5})} - (11+3\sqrt{5})(5+2\sqrt{5}) - \sqrt{15(5+2\sqrt{5})} \sqrt{6(25+11\sqrt{5})} + \right.$$

$$\left. + (5+2\sqrt{5}) \sqrt{6(25+11\sqrt{5})} \right) a^3 = \left[ \frac{\sqrt{3}}{9} \times (11+3\sqrt{5}) \sqrt{15(5+2\sqrt{5})} - \frac{\sqrt{3}}{9} (11+3\sqrt{5})(5+2\sqrt{5}) \right.$$

$$\left. - \frac{\sqrt{3}}{9} \sqrt{15(5+2\sqrt{5})} \sqrt{6(25+11\sqrt{5})} + \frac{\sqrt{3}}{9} (5+2\sqrt{5}) \sqrt{6(25+11\sqrt{5})} \right] a^3 =$$

$$[A - B - C + D] a^3; \quad \text{en donde}$$

$$A = \frac{\sqrt{3}}{9} (11+3\sqrt{5}) \sqrt{15(5+2\sqrt{5})} = \frac{\sqrt{3}}{9} \sqrt{15(5+2\sqrt{5})(11+3\sqrt{5})^2} =$$

$$= \frac{1}{9} \sqrt{45(5+2\sqrt{5})(121+45+66\sqrt{5})} = \frac{3}{9} \sqrt{5(5+2\sqrt{5})(166+66\sqrt{5})} =$$

$$= \frac{1}{3} \sqrt{5 \times 2 (5+2\sqrt{5})(83+33\sqrt{5})} = \frac{1}{3} \sqrt{10(415+166\sqrt{5}+165\sqrt{5}+330)} =$$

$$= \frac{1}{3} \sqrt{10(745+331\sqrt{5})} \quad \text{siendo } 331, \text{ primo } \text{ y } 745^2 - (331\sqrt{5})^2 = 7220$$

por lo que  $84^2 < 7220 < 85^2$ , no puede simplificarse más.

Igualmente

$$B = \frac{\sqrt{3}}{9} (11+3\sqrt{5})(5+2\sqrt{5}) = \frac{\sqrt{3}}{9} (55+15\sqrt{5}+22\sqrt{5}+30) = \frac{\sqrt{3}}{9} (85+37\sqrt{5}) =$$

$$= \frac{85\sqrt{3} + 37\sqrt{15}}{9}$$

1. The first part of the paper is devoted to a general discussion of the problem of the existence of solutions of the system of equations (1) and (2) under the assumption that the functions  $f_i(x)$  and  $g_j(x)$  are continuous and satisfy certain conditions. It is shown that if these conditions are satisfied, then the system has at least one solution in the domain  $D$ .

2. In the second part, we consider the case where the functions  $f_i(x)$  and  $g_j(x)$  are not continuous, but satisfy certain conditions. It is shown that in this case, the system may have no solutions or an infinite number of solutions.

3. The third part of the paper is devoted to a study of the stability of the solutions of the system. It is shown that if the functions  $f_i(x)$  and  $g_j(x)$  satisfy certain conditions, then the solutions are stable.

4. In the fourth part, we consider the case where the functions  $f_i(x)$  and  $g_j(x)$  are not continuous, but satisfy certain conditions. It is shown that in this case, the system may have no solutions or an infinite number of solutions.

5. The fifth part of the paper is devoted to a study of the stability of the solutions of the system. It is shown that if the functions  $f_i(x)$  and  $g_j(x)$  satisfy certain conditions, then the solutions are stable.

6. In the sixth part, we consider the case where the functions  $f_i(x)$  and  $g_j(x)$  are not continuous, but satisfy certain conditions. It is shown that in this case, the system may have no solutions or an infinite number of solutions.

7. The seventh part of the paper is devoted to a study of the stability of the solutions of the system. It is shown that if the functions  $f_i(x)$  and  $g_j(x)$  satisfy certain conditions, then the solutions are stable.

8. In the eighth part, we consider the case where the functions  $f_i(x)$  and  $g_j(x)$  are not continuous, but satisfy certain conditions. It is shown that in this case, the system may have no solutions or an infinite number of solutions.

9. The ninth part of the paper is devoted to a study of the stability of the solutions of the system. It is shown that if the functions  $f_i(x)$  and  $g_j(x)$  satisfy certain conditions, then the solutions are stable.

10. In the tenth part, we consider the case where the functions  $f_i(x)$  and  $g_j(x)$  are not continuous, but satisfy certain conditions. It is shown that in this case, the system may have no solutions or an infinite number of solutions.

Análogamente

$$C = \frac{\sqrt{3}}{9} \sqrt{15(5+2\sqrt{5})} \sqrt{6(25+11\sqrt{5})} = \frac{\sqrt{3}}{9} \times 3 \sqrt{10(5+2\sqrt{5})(25+11\sqrt{5})} =$$

$$= \frac{\sqrt{3}}{3} \sqrt{10(125+50\sqrt{5}+55\sqrt{5}+110)} = \frac{\sqrt{3}}{3} \sqrt{10(235+105\sqrt{5})} =$$

$$= \frac{1}{3} \sqrt{30 \times 5(47+21\sqrt{5})} = \frac{5}{3} \sqrt{6(47+21\sqrt{5})} = 9 \text{ siendo}$$

$$47^2 - (21\sqrt{5})^2 = 2^2, \quad \text{sea } = \frac{5}{3} \sqrt{6 \left( \sqrt{\frac{49}{2}} + \sqrt{\frac{45}{2}} \right)} = \frac{5\sqrt{6}}{3} \left( \frac{7}{\sqrt{2}} + \frac{3\sqrt{5}}{\sqrt{2}} \right) =$$

$$= \frac{5}{3} \left( 7\sqrt{\frac{6}{2}} + 3\sqrt{\frac{30}{2}} \right) = \frac{5}{3} (7\sqrt{3} + 3\sqrt{15}) = \frac{35\sqrt{3}}{3} + \frac{15\sqrt{15}}{3}$$

y finalmente

$$D = (5+2\sqrt{5}) \sqrt{6(25+11\sqrt{5})} \times \frac{\sqrt{3}}{9} = \frac{\sqrt{3}}{9} \sqrt{6(25+11\sqrt{5})(5+2\sqrt{5})^2} =$$

$$= \frac{\sqrt{3}}{9} \sqrt{6(25+11\sqrt{5})(25+20+20\sqrt{5})} = \frac{\sqrt{3}}{9} \sqrt{6(25+11\sqrt{5})(45+20\sqrt{5})} =$$

$$= \frac{\sqrt{3}}{9} \sqrt{30(25+11\sqrt{5})(9+4\sqrt{5})} = \frac{1}{9} \sqrt{90(225+99\sqrt{5}+100\sqrt{5}+220)} =$$

$$= \frac{1}{3} \sqrt{10(445+199\sqrt{5})} = (199 \text{ es primo y } 445^2 - (199\sqrt{5})^2 = 20$$

$$4^2 < 20 < 5^2)$$

De los valores anteriores se deduce la expresión final:

Main body of handwritten text, consisting of several lines of script. The text is written in a cursive style and appears to be a letter or a formal document. The lines are somewhat faded and difficult to read.



$$V_2 = (A + D - (B + C))a^3 = \left( \frac{\sqrt{10}(745 + 331\sqrt{5})}{3} + \frac{\sqrt{10}(445 + 199\sqrt{5})}{3} - \left( \frac{85\sqrt{3} + 37\sqrt{15}}{9} + \frac{35\sqrt{3} + 15\sqrt{15}}{3} \right) \right) a^3 = \left( \frac{\sqrt{10}(745 + 331\sqrt{5})}{3} + \frac{\sqrt{10}(445 + 199\sqrt{5})}{3} - \frac{85\sqrt{3} + 37\sqrt{15} + 105\sqrt{3} + 45\sqrt{15}}{9} \right) a^3 = \left( \frac{\sqrt{10}(745 + 331\sqrt{5})}{3} + \frac{\sqrt{10}(445 + 199\sqrt{5})}{3} - \frac{190\sqrt{3} + 82\sqrt{15}}{9} \right) a^3$$

el volumen total  $V$  del poliedro estudiado, será pues

$$V = V_1 + V_2 = \frac{10\sqrt{3} + 2\sqrt{15}}{9} a^3 + \left( \frac{\sqrt{10}(745 + 331\sqrt{5})}{3} + \frac{\sqrt{10}(445 + 199\sqrt{5})}{3} - \frac{190\sqrt{3} + 82\sqrt{15}}{9} \right) a^3 = \left( \frac{\sqrt{10}(745 + 331\sqrt{5})}{3} + \frac{\sqrt{10}(445 + 199\sqrt{5})}{3} + \frac{10(18\sqrt{3} + 8\sqrt{15})}{9} \right) a^3$$

Desarrollo del cálculo anterior:

$$V = V_1 + V_2 = \frac{10\sqrt{3} + 2\sqrt{15}}{9} a^3 + \left( \frac{\sqrt{10}(745 + 331\sqrt{5})}{3} + \frac{\sqrt{10}(445 + 199\sqrt{5})}{3} - \frac{190\sqrt{3} + 82\sqrt{15}}{9} \right) a^3 = \left( \frac{\sqrt{10}(745 + 331\sqrt{5})}{3} + \frac{\sqrt{10}(445 + 199\sqrt{5})}{3} + \frac{10\sqrt{3} + 2\sqrt{15} - 190\sqrt{3} - 82\sqrt{15}}{9} \right) a^3 = \left( \frac{\sqrt{10}(745 + 331\sqrt{5})}{3} + \frac{\sqrt{10}(445 + 199\sqrt{5})}{3} + \frac{10\sqrt{3} + 2\sqrt{15} - 190\sqrt{3} - 82\sqrt{15}}{9} \right) a^3$$

1. Introduction 2. Methodology 3. Results 4. Conclusion

The first part of the paper is devoted to the introduction of the problem.

The second part is devoted to the description of the method used.

The third part is devoted to the presentation of the results.

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The fourth part is devoted to the conclusion of the paper.

$$+ \frac{\sqrt{10} (445 + 199\sqrt{5})}{3} - \frac{180\sqrt{3} + 80\sqrt{15}}{9} \Big) a^3 =$$

$$= \left( \frac{\sqrt{10} (745 + 331\sqrt{5})}{3} + \frac{\sqrt{10} (445 + 199\sqrt{5})}{3} - \frac{10 (18\sqrt{3} + 8\sqrt{15})}{9} \right) a^3$$

Ángulo rectilíneo "20" del diedro formado en la intersección de una cara del dodecaedro, con la correspondiente del icosaedro

En la lámina 24, podemos comprobar que el ángulo formado por la cara 21-22-23-24-25 del dodecaedro regular, y la 1-5-6 del icosaedro regular, tiene dado en su verdadera magnitud en la proyección en I, ya que ambas caras son perpendiculares a dicho plano I.

Por consiguiente, el ángulo buscado 20 es, suplementario del que forma la cara triangular 1-5-6 con el plano horizontal II, el cual se deduce de la pirámide pentagonal que se forma en el vértice 1 del icosaedro, cuya altura  $\underline{g}$  y la apotema  $\underline{h}$  de su base son catetos de un triángulo rectángulo conocido (ver lám. 5, fórmulas 50 y 51)

Así pues, tendremos que

$$20 = \pi - \arctg \frac{g}{h} = \pi - \arctg \frac{\frac{\sqrt{5-\sqrt{5}}}{10} l_{20}}{\frac{\sqrt{5+2\sqrt{5}}}{20} l_{20}} =$$

The first part of the paper is devoted to a general discussion of the problem. It is shown that the problem is equivalent to the problem of finding a function  $f(x)$  which satisfies the conditions

$$f(x) = 0 \text{ for } x = 0, 1, 2, \dots, n$$

$$f(x) = 1 \text{ for } x = n+1, n+2, \dots, 2n$$

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The second part of the paper is devoted to a general discussion of the problem. It is shown that the problem is equivalent to the problem of finding a function  $f(x)$  which satisfies the conditions

$$= \pi - \arccos \left( \frac{1}{3} (3 - \sqrt{5}) \right)$$

Desarrollo del cálculo anterior:  $\frac{1}{2} \frac{g}{h} = \frac{1}{2} \frac{\frac{\sqrt{5-\sqrt{5}}}{10} l_{20}}{\frac{\sqrt{5+2\sqrt{5}}}{20} l_{20}} =$

$$= \frac{1}{2} \sqrt{\frac{5-\sqrt{5}}{10} : \frac{5+2\sqrt{5}}{20}} = \frac{1}{2} \sqrt{2(5-\sqrt{5}) : (5+2\sqrt{5})} = \frac{1}{2} \sqrt{\frac{2(5-\sqrt{5})}{5+2\sqrt{5}}} =$$

$$= \frac{1}{2} \sqrt{\frac{2(5-\sqrt{5})(5-2\sqrt{5})}{25-20}} = \frac{1}{2} \sqrt{\frac{2(25-5\sqrt{5}-10\sqrt{5}+10)}{5}} = \frac{1}{2} \sqrt{\frac{2(35-15\sqrt{5})}{5}} =$$

$$= \frac{1}{2} \sqrt{2(7-3\sqrt{5})} = \quad (\text{viendo } 7^2 - (3\sqrt{5})^2 = 2^2)$$

$$= \frac{1}{2} \sqrt{2} \left( \sqrt{\frac{7}{2}} - \sqrt{\frac{5}{2}} \right) = \boxed{\frac{1}{2} (3 - \sqrt{5})}$$

el cálculo numérico de "2θ" es el siguiente:

$$2\theta = \pi - \arccos \left( \frac{1}{3} (3 - \sqrt{5}) \right)$$

$$3 - \sqrt{5} \approx 0.763932$$

$$\frac{1}{3} 0.763932 = 0.254644$$

$$\arccos \left( \frac{1}{3} (3 - \sqrt{5}) \right) = 37^\circ 22' 38.5''$$

$$2\theta = 180^\circ - 37^\circ 22' 38.5'' = 142^\circ 37' 21.5''$$

Ángulo rectilíneo "2φ<sub>12</sub>" del diedro del dodecaedro

(ver lám. 4, fórm. 34)

$$2\varphi_{12} = 116^\circ 33' 54.2''$$

$$\cos \varphi_{12} = \sqrt{\frac{5+\sqrt{5}}{10}} = 0.850651...$$



1. The first part of the paper is devoted to a general discussion of the problem.

2. In the second part, we consider the case of a single particle.

3. The third part is devoted to the case of a system of particles.

4. In the fourth part, we consider the case of a continuous medium.

5. The fifth part is devoted to the case of a system of continuous media.

6. In the sixth part, we consider the case of a single continuous medium.

7. The seventh part is devoted to the case of a system of continuous media.

8. In the eighth part, we consider the case of a single continuous medium.

9. The ninth part is devoted to the case of a system of continuous media.

Ángulo rectilíneo " $2\varphi_{20}$ " del diedro del icosaedro

(ver lám. 5, fórm. 47)

$$2\varphi_{20}'' = 138^\circ 11' 22,8''$$

$$\operatorname{sen} \varphi_{20} = \frac{\sqrt{15} + \sqrt{3}}{6} = 0,93 \ 41 \ 72 \dots$$

CUADRO SINÓPTICO

Magnitud	Valor exacto	Valor decimal aproximado
<sup>226</sup> $l_{12}$	$\frac{\sqrt{15} - \sqrt{3}}{3} a$	0,71 36 44...a
<sup>227</sup> $l_{20}$	$\frac{\sqrt{10} \times (5 - \sqrt{5})}{5} a$	1,05 14 62...a
<sup>229</sup> $f$	$\frac{3\sqrt{10} + 2\sqrt{5} - (3\sqrt{3} + \sqrt{15})}{6} a$	0,39 05 90...a
<sup>229</sup> $i$	$\left(\frac{\sqrt{5} + 1}{2} - \sqrt{\frac{25 + 11\sqrt{5}}{30}}\right) a$	0,33 22 56...a
<sup>230</sup> $h$	$\left(1 - \sqrt{\frac{5 + 2\sqrt{5}}{15}}\right) a$	0,20 53 46...a
<sup>231</sup> $S$	$5 \left(18\sqrt{3} + 6\sqrt{5} - \sqrt{10(25 + 11\sqrt{5})} - 3\sqrt{2(25 + 11\sqrt{5})}\right) a^2$	11,32 85 41...a <sup>2</sup>
<sup>232</sup> $V$	$\left(\frac{\sqrt{10}(745 + 331\sqrt{5})}{3} + \frac{\sqrt{10}(445 + 199\sqrt{5})}{3} - \frac{10(18\sqrt{3} + 8\sqrt{5})}{9}\right) a^3$	3,00 07 58...a <sup>3</sup>
<sup>233</sup> $2\theta$	$2\theta = \pi - \operatorname{arc} \operatorname{tg} (3 - \sqrt{5})$	142° 37' 21,5''
<sup>234</sup> $2\varphi_{12}$	$\operatorname{sen} \varphi_{12} = \sqrt{\frac{5 + \sqrt{5}}{10}}$	$\operatorname{sen} \varphi_{12} = 0,85 \ 06 \ 51$ $2\varphi_{12} = 116^\circ 33' 54,2''$
<sup>235</sup> $2\varphi_{20}$	$\operatorname{sen} \varphi_{20} = \frac{\sqrt{15} + \sqrt{3}}{6}$	$\operatorname{sen} \varphi_{20} = 0,93 \ 41 \ 72$ $2\varphi_{20} = 138^\circ 11' 22,8''$



FIGURA CORPÓREA

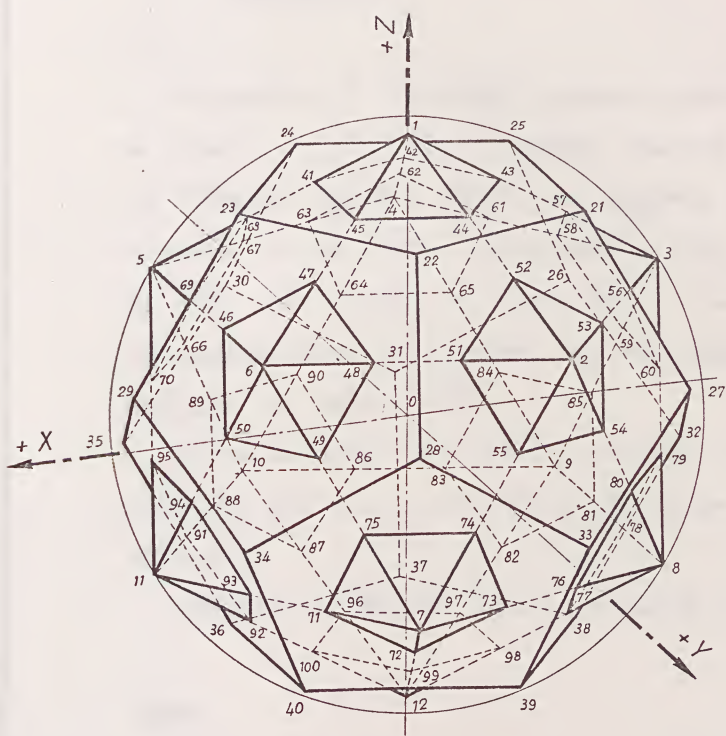
Para obtener esta figura, se construye previamente el dodecaedro regular de 39,3 mm de lado, en cuyas caras se dibujará previamente la base de la pirámide complementaria correspondiente al icosaedro. Dicha base es un pentágono regular de 21,5 mm de lado, siendo el radio de su circunferencia circunscrita de 18,3 mm.; este pentágono está colocado en la cara del dodecaedro con su centro coincidente y sus vértices en las mediatrices de los lados de ésta.

Posteriormente se acoplarán a las caras del dodecaedro las doce pirámides formadas cada una por cinco triángulos equiláteros de 21,5 mm de lado.



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*Poliedros regulares convexos conjugados*



Following the construction of the crystallographic form, the next step is to determine the relative positions of the various faces of the crystal.

## ENUNCIADO

Representar por el método gráfico-analítico, en los planos I, II, III, el poliedro derivado de un tetraedro regular, obtenido al proyectar desde el centro de la esfera circunscrita a éste, y sobre ella, los centros de cada cara, uniendo a continuación estos puntos con los vértices del polígono de dicha cara.

Las coordenadas del centro de la esfera son  $O(72, 72, 85)$  mm y el radio de la esfera es de 55 mm.

Dibujar en formato A3 y a escala 1:1.

DATOS

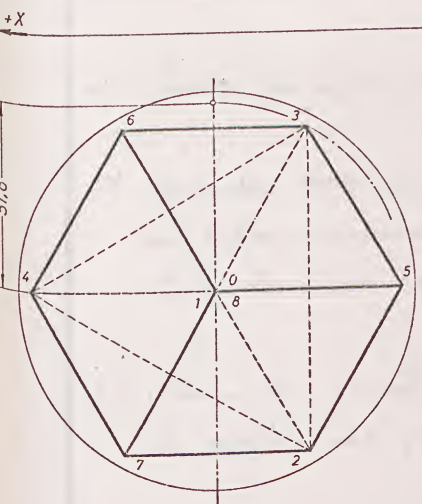
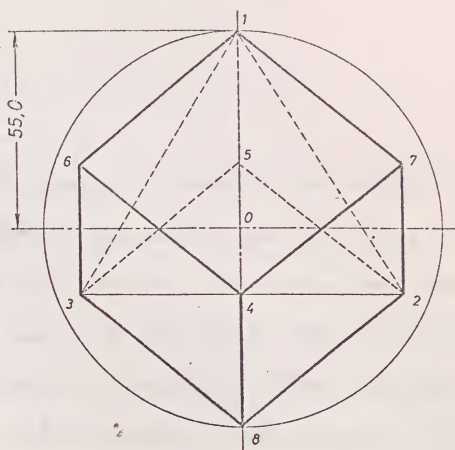
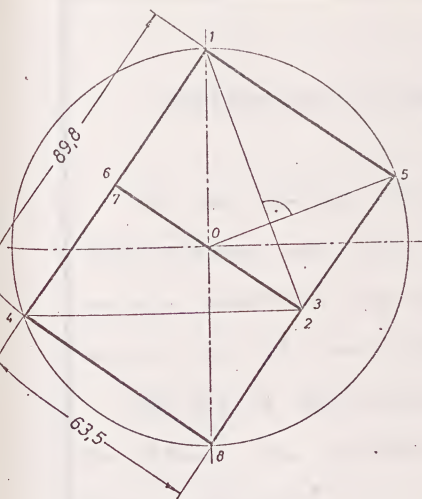
$O(72, 72, 85)$  mm

$a_4 = 55$  mm.

100

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100 100 100



NUMERACIÓN DE VÉRTICES

Tetraedro..... 1 al 4

Poliedro derivado (exaedro)----- 1 al 8

### ENUNCIADO

Representar por el método gráfico-analítico, en los planos I, II y III el poliedro derivado de un tetraedro regular obtenido al proyectar desde el centro de la esfera circunscrita a éste, y sobre ella, los centros de cada cara, uniendo a continuación estos puntos con los vértices del polígono de dicha cara.

Las coordenadas del centro de la esfera son:  $O(72, 72, 85)$  mm y el radio de la misma de 55 mm.

Dibujar en formato A3v y a escala 1:1.

	Propuesta	De entrega	Entregada	Califi- cación	(firma)	Escuela
Fecha:						Curso
Alumno:						
Escala 1:1	Poliedro derivado del tetraedro regular					Lámina 25 Curso 10





CONSIDERACIONES PREVIAS

En las láminas 25 a 30 vamos a realizar el estudio y representación de los poliedros derivados de los cinco regulares convexos, siguiendo la ley de formación que se expresa en el enunciado para el derivado del Tetraedro.

Esta ley de formación, común para todos ellos, consiste en obtener un poliedro de otro regular dado según el proceso siguiente:

- 1º) Trazar la esfera circunscrita al poliedro regular dado.
- 2º) Proyectar desde el centro de ésta, y sobre su esfera circunscrita, los centros de los polígonos de todas las caras de dicho poliedro regular dado.
- 3º) Unir los puntos obtenidos en el proceso 2º, con los vértices del polígono de su cara correspondiente.

De la ley de formación expuesta, se deducen la siguientes propiedades comunes a los cinco poliedros derivados:

- a) Todas las caras del poliedro derivado son iguales, y tienen forma de triángulos isósceles o rombos.



En efecto, al unir los puntos obtenidos por el proceso 2.º con los vértices del polígono de cada cara del poliedro regular dado, se forma en cada una de ellas una pirámide recta regular de tantas caras laterales como lados tenga dicha polígono. Todas las pirámides serán iguales y por consiguiente todas las caras del poliedro derivado serán a su vez iguales.

Existe la posibilidad de que las caras adyacentes de dos pirámides contiguas con arista común en una arista del poliedro regular dado, puedan o no estar en un mismo plano. En el primer caso, cada dos caras triangulares contiguas sobre la arista del poliedro regular, se convierten en una sola cara con la forma de rombo, ya que sus cuatro lados serían iguales.

Si llamamos  $C'_n$ ,  $V'_n$  y  $A'_n$  el número de caras, vértices y aristas del poliedro regular dado, y  $C$ ,  $V$  y  $A$  los del poliedro derivado de igual esfera circumsrita, se verificará, bajo el supuesto de ser todas sus caras triangulares, que

$$C = n C'_n$$

[1]

puesto que cada cara del poliedro regular dado, es base de una pirámide recta regular de "n" caras laterales, siendo  $n$  el número de lados del polígono de su base.

También tendremos que

The following is a list of the names of the members of the American Medical Association who have been elected to the office of President for the year 1917. The names are listed in alphabetical order of their last names.

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$$V = V'_n + C'_n \quad [2]$$

ya que a más de los vértices del poliedro dado, se originan nuevos vértices, uno por cada cara de dicho poliedro.

Y finalmente se verificará que

$$A = A'_n + n C'_n \quad [3]$$

puesto que a más de las aristas del poliedro dado, se originan nuevas aristas en las caras laterales de las pirámides que se forman en cada cara del mismo, cuyo número es igual al de lados del polígono base de dicha pirámide.

Sumando las igualdades [1], [2], y restando la [3], tendremos

$$C + V - A = n C'_n + V'_n + C'_n - A'_n - n C'_n = C'_n + V'_n - A'_n$$

y aplicando el teorema de Euler al poliedro convexo dado, se verificará que

$$C'_n + V'_n - A'_n = 2$$

por lo que en el poliedro derivado también se verificará dicho teorema, siendo pues

$$C + V - A = 2$$



aun cuando este poliedro derivado no sea convexo, como veremos ocurre en algunos casos,

- b) Existe una esfera, concéntrica con la circunscrita al poliedro derivado, que es tangente en un punto medio a todas las aristas de igual longitud

En efecto, según la ley de formación de estos poliedros derivados, todos los vértices del mismo se encuentran en una misma esfera que es la circunscrita al poliedro regular dado, y por consiguiente también circunscrita al poliedro derivado.

Cualquier arista de este poliedro tiene sus extremos  $A$  y  $B$  en dicha esfera de radio  $a$  (fig. 1).

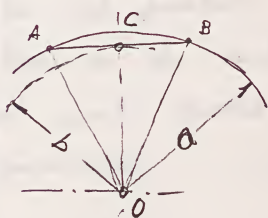


Figura 1

y centro  $O$ . Si unimos  $A$  y  $B$  con  $O$ , el triángulo  $AOB$  será isósceles y el punto, medio  $C$  de  $AB$  será el pie de la perpendicular trazada a  $AB$  desde  $O$  (altura del triángulo  $AOB$ , correspondiente al lado  $AB$ ).

Si consideramos que la arista  $AB$  se mueve sobre la esfera de forma que sus extremos estén siempre sobre ella, el punto  $C$  equidistará siempre de  $O$  la magnitud  $OC = b$  y por consiguiente el l.g. de sus infinitas posiciones será una es-

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fera de centro 0 y radio b.

En todo poliedro derivado existen dos clases de aristas de diferentes magnitudes entre sí, pero iguales en cada clase, que son las siguientes:

La primera clase comprenden las aristas que forman el contorno de cada pirámide recta cuya base es el polígono regular de cada cara del poliedro regular dado; la segunda clase son las aristas laterales de dichas pirámides.

Así pues, en todo poliedro derivado, existirá una esfera de radio b<sub>1</sub>, concéntrica con la circunscrita, que será tangente en su punto medio a las aristas del poliedro regular dado, y otra esfera análoga a la anterior, de radio b<sub>2</sub> tangente en su punto medio a las aristas laterales de la pirámide.

c) Existe una esfera, concéntrica con la circunscrita al poliedro derivado, que es tangente a las caras de éste (inscrita en el mismo).

En efecto, todas las caras del poliedro derivado son iguales, pudiendo ser triángulos isósceles o rombos (ver propiedad a). Ambas figuras tienen al menos un eje de simetría y sus vértices en la esfera





sea circunscrita, lo cual nos demuestra que al aplicar  
 a una cara cualquiera un giro con centro en  $O$ ,  
 podemos obtener siempre la superposición de dicha  
 cara con otra del poliedro derivado. El ortocentro del  
 triángulo de cada cara permanecerá equidistante de  $O$   
 durante todo el movimiento y será el punto de con-  
 tacto de la esfera tangente inscrita en el poliedro  
 derivado.

(sigue en hoja 7)

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CALCULO DE LOS ANGULOS RECTILÍNEOS DE LOS DIEDROS DE LAS  
CARAS DEL POLIEDRO DERIVADO Y LONGITUDES FUNDAMENTALES

En el poliedro derivado sólo existen dos clases de diedros, iguales entre sí en cada clase; éstas son las siguientes:

- a) Diedros que se forman en dos caras triangulares contiguas con arista común a las del poliedro regular dado, 2
- b) Diedros que se forman en dos caras triangulares contiguas en las aristas de las pirámides rectas regulares que se forman en cada cara de dicho poliedro regular dado.

Para el desarrollo de los cálculos anteriores, emplearemos la siguiente nomenclatura, análoga a la de las láminas anteriores:

$n$  = Número de caras del poliedro regular dado

$a_n$  = Radio de la esfera circunscrita a dicho poliedro.

$l_n$  = Lado del poliedro regular dado

$b_1$  = Radio de la esfera tangente a las aristas del poliedro regular dado

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- (16)  $b_2 =$  Radio de esfera tangente a las aristas <sup>(laterales)</sup> de las pirámides rectas cuyas bases son caras del mismo.
- (15)  $c_n =$  Radio de la esfera inscrita en el poliedro regular dado
- (16)  $c_1 =$  Radio de la esfera inscrita en el poliedro derivado.
- (16)  $d_n =$  Radio de la circunferencia circunscrita al polígono regular de una cara del poliedro dado.
- (17)  $k_n =$  Apotema del polígono regular del mismo, o también radio de su circunferencia circunscrita.
- (18)  $2\varphi_n =$  Diedro del poliedro regular dado
- (19)  $2\alpha_n =$  Diedro de dos caras del poliedro derivado en la arista del poliedro dado.
- (16)  $\beta_n =$  Diedro formado por una cara lateral de la pirámide, y su base.
- (13)  $2\gamma_n =$  Diedro de dos caras de la pirámide.
- (16)  $p =$  Altura de una cara lateral de la pirámide recta formada en cada cara del mismo.
- (11)  $q =$  Arista lateral de dicha pirámide o lado igual de una cara triangular isósceles
- (12)  $t =$  Distancia entre extremos de dos lados consecutivos del polígono de una cara del poliedro dado.
- (17)  $S =$  Superficie del poliedro derivado
- (18)  $V =$  Volumen del poliedro derivado



- a) Cálculo del diedro que forman dos caras triangulares del poliedro derivado, con arista común a la del poliedro regular dado.

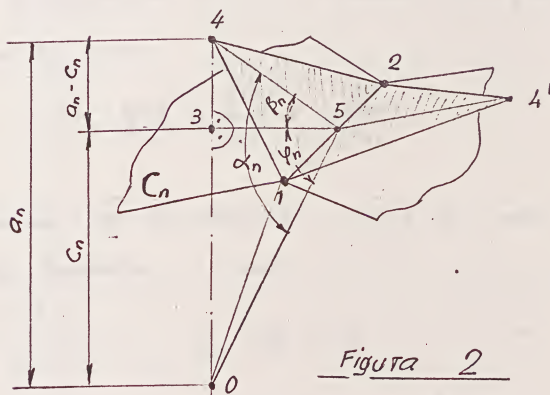


Figura 2

Sea (fig. 2) 1-2 una arista del poliedro regular dado, O su centro y 3 el centro de una cara C<sub>n</sub> de dicho poliedro.

El punto 4, obtenido al proyectar desde O sobre (véase en hoja 10)

The first of these is the fact that the  
bones of the human skeleton are  
not all of the same size and shape.



Another fact is that the bones of the  
human skeleton are not all of the same  
color and texture. Some are white and  
smooth, while others are yellow and  
rough. This is due to the fact that the  
bones of the human skeleton are not all  
of the same age and condition.

la esfera circunscrita, el centro 3 de la cara  $C_n$ , será el vértice de una de las pirámides rectas regulares de  $n$  caras laterales. Sea  $\underline{5}$  el punto, medio de la arista 1-2, que uniremos 3 y 4, formando el triángulo 3-4-5, rectángulo en 3. En este triángulo se verificará que

$$\frac{1}{2} \beta = \frac{3-4}{3-5} = \frac{a_n - c_n}{k_n}$$

En el triángulo 0-3-5, también rectángulo en 3, se verificará a su vez

$$\frac{1}{2} \varphi_n = \frac{3-0}{3-5} = \frac{c_n}{k_n}$$

La suma de los ángulos  $\varphi_n$  y  $\beta$ , será igual al ángulo  $\alpha_n$  buscado, o sea

$$\alpha_n = \varphi_n + \beta$$

y de las expresiones anteriores se obtiene

$$\begin{aligned} \frac{1}{2} \alpha_n &= \frac{1}{2} (\varphi_n + \beta) = \frac{\frac{1}{2} \varphi_n + \frac{1}{2} \beta}{1 - \frac{1}{2} \varphi_n - \frac{1}{2} \beta} = \frac{\frac{c_n}{k_n} + \frac{a_n - c_n}{k_n}}{1 - \frac{c_n}{k_n} - \frac{a_n - c_n}{k_n}} = \\ &= \frac{a_n k_n}{k^2 - a_n c_n + c_n^2} \quad [4] \end{aligned}$$

con cuyo valor se puede determinar, en cada caso particular, el del ángulo  $\alpha_n$ .

Este ángulo define la característica especial del



I have the honor to acknowledge the receipt of your letter of the 10th inst. in relation to the above matter. The same has been forwarded to the proper authorities for their consideration. I am, Sir, very respectfully,  
 Yours very truly,  
 J. H. [Signature]

- 1 - 10

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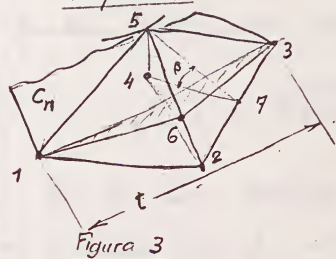
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poliedro derivado, el cual será cóncavo o convexo según sea  $\alpha_n$  menor o mayor de un recto.

En el caso particular de que  $\alpha_n$  sea recto, el poliedro derivado tendrá dos caras laterales en un mismo plano, es decir, se transformarán éstas en un paralelogramo de lados iguales (rombo). El número de caras de este poliedro, determinado por la expresión [1], se reducirá a la unidad; el de vértices por la expresión [2] será el mismo y el de aristas por la expresión [3], quedará reducida a  $A_n = n C_n$ , por desaparecer las aristas  $A'_n$  (son diagonales del rombo).

b) Cálculo del diedro que se forma en dos caras laterales contiguas en las aristas de la pirámide recta regular.



Sea (fig. 3)  $C_n$  una cara del poliedro regular dado, de  $n$  lados; 4 el centro de la misma y 5 el vértice de la pirámide recta regular de  $n$  caras laterales

que se forma en  $C_n$ . Dos lados consecutivos 1-2 y 2-3 de  $C_n$  unidos sus extremos con 5, nos determinarán dos caras laterales contiguas 1-5-2 y 2-5-3, de arista común 5-2.

The first part of the paper is devoted to a discussion of the  
 general principles of the theory of the structure of the  
 crystal lattice. It is shown that the structure of the  
 crystal lattice is determined by the nature of the  
 chemical bonds between the atoms. The structure of the  
 crystal lattice is also determined by the nature of the  
 external forces acting on the crystal. The structure of the  
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El rectilíneo de este diedro se puede obtener por intersección de un plano que pasando por los extremos 1 y 3 de los dos lados consecutivos 1-2 y 2-3, sea perpendicular a la arista 5-2.

Sea 6 el punto de intersección de dichos planos con 5-2; uniendo 6 con 1 y 3 se nos formará el triángulo isósceles 1-6-3, cuyo ángulo en 6 será el pedido  $2\gamma_n$ .

Para obtener su valor, unamos el punto 7, medio del lado 2-3, con el centro 4, formándose el triángulo 5-4-7, rectángulo en 4 por lo que se verificará que

$$p = 5-7 = \sqrt{(4-5)^2 + (4-7)^2} = \sqrt{(a_n - c_n)^2 + k_n^2} \quad [5]$$

Uniendo 4 con 2, se nos formará otro triángulo 5-4-2, también rectángulo en 4, por lo que

$$q = 5-2 = \sqrt{(4-5)^2 + (4-2)^2} = \sqrt{(a_n - c_n)^2 + d_n^2} \quad [6]$$

En la cara triangular 5-2-3 se puede determinar el ángulo en 2 en función de p y q, ya que se verificará que

$$\cos(\widehat{5-2-7}) = \cos(\widehat{2-5-7}) = \frac{5-7}{5-2} = \frac{p}{q}$$

En el triángulo 2-6-3, rectángulo en 6, se verificará que

The function  $f(x) = \frac{1}{2} \log \frac{1+x}{1-x}$  is defined for  $x \in (-1, 1)$ . It is an odd function, i.e.,  $f(-x) = -f(x)$ . The function is strictly increasing on  $(-1, 1)$ . The range of  $f(x)$  is  $(-\infty, \infty)$ . The function has a vertical asymptote at  $x = 1$  and a horizontal asymptote at  $y = 0$ . The function is concave down on  $(-1, 1)$ . The function is a bijection from  $(-1, 1)$  to  $(-\infty, \infty)$ . The function is the inverse of the function  $g(y) = \frac{e^y - 1}{e^y + 1}$ .

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(1)  $\frac{1}{2} \log \frac{1+x}{1-x} = \frac{1}{2} \log \frac{1+x}{1-x}$   
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 The function  $f(x) = \frac{1}{2} \log \frac{1+x}{1-x}$  is defined for  $x \in (-1, 1)$ . It is an odd function, i.e.,  $f(-x) = -f(x)$ . The function is strictly increasing on  $(-1, 1)$ . The range of  $f(x)$  is  $(-\infty, \infty)$ . The function has a vertical asymptote at  $x = 1$  and a horizontal asymptote at  $y = 0$ . The function is concave down on  $(-1, 1)$ . The function is a bijection from  $(-1, 1)$  to  $(-\infty, \infty)$ . The function is the inverse of the function  $g(y) = \frac{e^y - 1}{e^y + 1}$ .

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$$3 \cdot 6 = (2 \cdot 3) \times \text{sen}(\widehat{6 \cdot 2 \cdot 3}) = l_n \times \text{sen}(\widehat{5 \cdot 2 \cdot 7}) = l_n \times \frac{p}{q}$$

Consideremos finalmente el triángulo isósceles 1.6.3, en el que designaremos por  $\frac{1}{2}$  al lado 1.3. El valor del diedro  $2\gamma_n$  buscado se deduce de la expresión

$$\text{sen } \gamma_n = \cos(\widehat{1 \cdot 3 \cdot 6}) = \frac{\frac{1}{2}(1 \cdot 3)}{3 \cdot 6} = \frac{\frac{1}{2}}{l_n \times \frac{p}{q}} = \frac{tq}{2 l_n p} \quad [7]$$

c) Cálculo del diedro  $\beta_n$  formado por una cara lateral de la pirámide y su base.

Refiriéndonos a la fig. 3, vemos que

$$\text{sen } \beta_n = \text{sen}(\widehat{5 \cdot 7 \cdot 4}) = \cos(\widehat{4 \cdot 5 \cdot 7}) = \frac{4 \cdot 5}{5 \cdot 7} = \frac{a_n - c_n}{p} \quad [8]$$

Como comprobación de los cálculos anteriores, deberá verificarse (ver fig. 2) que

$$\alpha_n = \varphi_n + \beta_n$$

d) Cálculo del radio  $b_2$  de la esfera <sup>1</sup>tangente a las aristas de la pirámide.

En la figura 2 podemos observar que los puntos 1 y 4, extremos de una arista q de la pirámide, están sobre la esfera circunscrita de radio  $a_n$ ; por

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consecuente el triángulo 1-0-4 será isósceles y su altura desde 0 será perpendicular a la base 4-1 = 9 en su punto medio. Dicha altura será pues el radio  $b_2$  de la esfera pedida; su valor se deduce del triángulo rectángulo de cateto  $\frac{9}{2}$  e hipotenusa  $a_n$ , por lo que

$$b_2 = \sqrt{(a_n)^2 - \frac{9^2}{4}} \quad [9]$$

e) Cálculo del radio  $c_1$  de la esfera inscrita en el poliedro derivado

En la figura 2' podemos observar que si 1-2 es la arista común a dos caras triangulares que forman entre sí el ángulo  $2\alpha_n$  los vértices 4, 4' de dichas caras están sobre la esfera circunscrita de radio  $a_n$ ; por otra parte, en el triángulo 4-0-5 se verifica que  $0-5 = b_1$ , y la altura sobre 4-5 desde 0, o sea el radio  $c_1$  pedido, valdrá

$$c_1 = (0.5) \times \text{sen } \alpha_n = b_1 \text{ sen } \alpha_n \quad [10]$$

También podemos obtener el valor anterior operando con una arista común a dos caras laterales de la pirámide (p.e. la 4-1), que forman entre sí el ángulo  $2\gamma_n$ , por lo que

$$c_1 = b_2 \text{ sen } \gamma_n \quad [10']$$

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debiendo verificarse como comprobación que

$$b_1 \cos \alpha_n = b_2 \cos \gamma_n \quad [11]$$

### Resumen final

Las fórmulas [4] a [10] (o 10') junto con la [11] de comprobación, nos permiten calcular los elementos fundamentales de los poliedros derivados de los cinco poliedros regulares, obtenidos por el proceso geométrico detallado al comienzo de este estudio.

Dichas fórmulas son función de los valores  $2\varphi_n$ ,  $l_n$ ,  $b_n$ ,  $c_n$ ,  $d_n$ ,  $k_n$  y  $t$ , que deberán determinarse previamente para cada caso particular, en función del radio  $a_n$  del poliedro regular aplicado, único dato del problema.

El estudio se completará con el cálculo del área lateral  $S$  y volumen  $V$  del poliedro derivado.

Este será el proceso a seguir en el estudio de cada uno de los cinco poliedros derivados de los regulares.





DERIVADO DEL TETRAEDRO REGULARPROCESO GRÁFICO

Según se deduce de las consideraciones anteriores, para la representación gráfica de cualquier poliedro derivado de uno regular dado según el proceso indicado en el enunciado, basta con representar previamente el poliedro regular dado del ejercicio, y a continuación determinar los vértices de su conjugado inscrito en la misma esfera que aquél, colocado de tal forma que resulten dichos vértices alineados con el centro de una cara y el centro del poliedro.

En el caso particular que nos ocupa, el proceso gráfico es inmediato, ya que el conjugado del tetraedro regular es otro tetraedro también regular, y esta representación ha sido ya resuelta en el ejercicio de la lámina 22, cuyo proceso nos permite:

- 1° Representar el tetraedro regular dado, de vértices 1 al 4, inscrito en una esfera de 55 mm de radio.
- 2° Obtener los vértices del tetraedro conjugado 5 al 8, inscrito en la misma esfera (estos vértices se han de corresponder con los 11 al 14 de la lámina 22).
- 3° Unir los vértices 5 al 8 con los correspondientes de cada cara del tetraedro dado.

Al terminar la representación del poliedro derivado,



podemos observar que dos caras triangulares de éste, que sean contiguas y coincidentes con una arista del tetraedro dado, están en el mismo plano, por lo que al aplicar la fórmula [1] para la determinación del número de caras del derivado

$$C_4 = 3 \times C'_4 = 3 \times 4 = 12$$

este número se reduce a la mitad, y por lo tanto este poliedro tendrá 6 caras.

Por otra parte también podemos deducir de la representación final que los ángulos de estas seis caras, son rectos, y por consiguiente dichas caras (rectángulos) serán cuadradas.

De ello se deduce que "El poliedro derivado del tetraedro regular, es un hexaedro regular"

La demostración de esta propiedad la hacemos a continuación analíticamente en el estudio del proceso gráfico-analítico aplicado a este ejercicio.

#### PROCESO GRÁFICO-ANALÍTICO

Calculemos previamente los siguientes valores deducidos en láminas anteriores, en función del radio  $a_4$  (dato) de la esfera circunscrita al tetraedro regular dado.

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Número de caras "n" del tetraedro dado

$$n = 4$$

Radio "a<sub>4</sub>" de la esfera circunscrita al mismo (dato del ejercicio).Lado "l<sub>4</sub>" del tetraedro dado

Se deduce de la fórmula 1, lám. 1

$$l_4 = \frac{4}{\sqrt{6}} a_4 = \frac{2\sqrt{6}}{3} a_4$$

Radio "c<sub>4</sub>" de la esfera inscrita al mismo

Se deduce de la fórmula 3, lám. 1

$$c_4 = \frac{\sqrt{6}}{12} l_4 = \frac{\sqrt{6}}{12} \times \frac{2\sqrt{6}}{3} a_4 = \frac{1}{3} a_4$$

Radio "d<sub>4</sub>" de la circunferencia circunscrita al polígono regular de una cara del mismo.

Se deduce de la fórmula 4, lám. 1

$$d_4 = \frac{\sqrt{3}}{3} l_4 = \frac{\sqrt{3}}{3} \times \frac{2\sqrt{6}}{3} a_4 = \frac{2\sqrt{2}}{3} a_4$$

Radio "h<sub>4</sub>" de la circunferencia inscrita al polígono regular de una cara del mismo (apotema)

Se deduce de la fórmula 6, lám. 1

10

1000

1000

1000

1000

1000

1000

1000

1000

1000

1000

1000

1000

1000

1000

$$k_4 = \frac{\sqrt{3}}{6} l_4 = \frac{\sqrt{3}}{6} \times \frac{2\sqrt{6}}{3} a_4 = \frac{\sqrt{2}}{3} a_4$$

Radio "b<sub>1</sub>" de la esfera tangente a las aristas del poliedro regular dado.

Se deduce de la fórmula 2, lám. 1

$$b_1 = b_4 = \frac{\sqrt{2}}{4} l_4 = \frac{\sqrt{2}}{4} \times \frac{2\sqrt{6}}{3} a_4 = \frac{\sqrt{3}}{3} a_4$$

(sigue en hoja 20)



Angulo rectilíneo " $2\varphi_4$ " del diedro del mismo

Se deduce de la fórmula 5, lám. 1

$$\text{sen } \varphi_4 = \frac{\sqrt{3}}{3} \quad \underline{2\varphi_4 = 70^\circ 31' 43,6''}$$

Tomando como base los valores anteriores, deduciremos los siguientes del poliedro derivado.

Angulo rectilíneo " $2\alpha_4$ " del diedro formado por dos caras contiguas del poliedro derivado, en una arista del tetraedro dado.

Se deduce de la fórmula general [4] obtenida anteriormente (ver consideraciones previas) sustituyendo en ella los valores particulares de este caso

$$\text{tg } \alpha_4 = \frac{a_4 k_4}{(k_4)^2 - a_4 c_4 + (c_4)^2} = \frac{a_4 \times \frac{\sqrt{2}}{3} a_4}{\left(\frac{\sqrt{2}}{3} a_4\right)^2 - a_4 \times \frac{1}{3} a_4 + \left(\frac{1}{3} a_4\right)^2} = \frac{\frac{1}{3} \sqrt{2}}{0} = \infty$$

Desarrollo del cálculo anterior :

$$\begin{aligned} \text{tg } \alpha_4 &= \frac{a_4 \times \frac{\sqrt{2}}{3} a_4}{\left(\frac{\sqrt{2}}{3} a_4\right)^2 - a_4 \times \frac{1}{3} a_4 + \left(\frac{1}{3} a_4\right)^2} = \frac{\frac{\sqrt{2}}{3} (a_4)^2}{\frac{2}{9} (a_4)^2 - \frac{1}{3} (a_4)^2 + \frac{1}{9} (a_4)^2} = \\ &= \frac{\frac{\sqrt{2}}{3}}{\frac{2}{9} - \frac{1}{3} + \frac{1}{9}} = \boxed{\frac{\frac{\sqrt{2}}{3}}{0} = \infty} \end{aligned}$$



The first part of the paper is devoted to a discussion of the  
 various methods of determining the rate of reaction.

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 various methods of determining the rate of reaction.

The tenth part of the paper is devoted to a discussion of the  
 various methods of determining the rate of reaction.

Del valor anterior se deduce

$$\varphi_h = \arctg \infty = \frac{\pi}{2} = 90^\circ$$

y por consiguiente

$$2 \varphi_h = \pi = 180^\circ$$

lo que nos demuestra que las dos caras triangulares consideradas están en un mismo plano y se transforman en un rombo.

Altura "p" de una cara lateral de la pirámide recta formada en cada cara del tetraedro dado (cara del poliedro derivado).

Se deduce de la fórmula general [5] obtenida anteriormente (ver consideraciones previas) sustituyendo en ella los valores particulares de este caso

$$p = \sqrt{(a_4 - c_4)^2 + (k_4)^2} = \sqrt{\left(a_4 - \frac{1}{3} a_4\right)^2 + \left(\frac{\sqrt{2}}{3} a_4\right)^2} = \frac{\sqrt{6}}{3} a_4$$

Desarrollo del cálculo anterior:  $p = \sqrt{\left(a_4 - \frac{a_4}{3}\right)^2 + \left(\frac{\sqrt{2}}{3} a_4\right)^2} =$

$$= \sqrt{\left(\frac{2a_4}{3}\right)^2 + \left(\frac{\sqrt{2}}{3} a_4\right)^2} = \sqrt{\frac{4}{9} (a_4)^2 + \frac{2}{9} (a_4)^2} = \sqrt{\frac{6}{9} a_4^2} = \boxed{\frac{\sqrt{6}}{3} a_4}$$





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Arista lateral "q" de la pirámide recta regular, o lado igual del triángulo isósceles de una cara del poliedro derivado

Se deduce de la fórmula general [6] obtenida anteriormente (ver consideraciones previas), substituyendo en ella los valores particulares de este caso.

$$q = \sqrt{(a_4 - c_4)^2 + d_4^2} = \sqrt{\left(a_4 - \frac{1}{3}a_4\right)^2 + \left(\frac{2\sqrt{2}}{3}a_4\right)^2} = \frac{2\sqrt{3}}{3} a_4$$

$$\text{Desarrollo del cálculo anterior: } q = \sqrt{\left(a_4 - \frac{1}{3}a_4\right)^2 + \left(\frac{2\sqrt{2}}{3}a_4\right)^2} =$$

$$= \sqrt{\left(\frac{2a_4}{3}\right)^2 + \left(\frac{2\sqrt{2}}{3}a_4\right)^2} = \sqrt{\frac{4}{9}a_4^2 + \frac{8}{9}a_4^2} = \sqrt{\frac{12}{9}} a_4 = \boxed{\frac{2\sqrt{3}}{3} a_4}$$

Comparando los valores de  $l_4$  y  $q$ , deducidos en este estudio, base y lado del triángulo isósceles de una cara del poliedro derivado, vemos que siendo

$$l_4 = \frac{2\sqrt{6}}{3} a_4 \quad \text{y} \quad q = \frac{2\sqrt{3}}{3} a_4, \quad \text{se deduce que}$$

$$\frac{l_4}{q} = \frac{2\sqrt{6}}{3} a_4 : \frac{2\sqrt{3}}{3} a_4 = \sqrt{2} \quad \text{lo cual nos de-}$$

muestra que dicho triángulo isósceles es rectángulo en el vértice opuesto a la base.

Como por otra parte hemos visto en el cálculo

The first part of the paper is devoted to the study of the properties of the function  $f(x)$  defined by the equation

$$f(x) = \frac{1}{2} \left( f\left(\frac{x}{2}\right) + f\left(\frac{x+1}{2}\right) \right)$$

$$f(0) = 0, f(1) = 1$$

$$f\left(\frac{1}{2}\right) = \frac{1}{2}$$

$$f\left(\frac{1}{4}\right) = \frac{1}{4}, f\left(\frac{3}{4}\right) = \frac{3}{4}$$

The second part of the paper is devoted to the study of the properties of the function  $f(x)$  defined by the equation

$$f(x) = \frac{1}{2} \left( f\left(\frac{x}{2}\right) + f\left(\frac{x+1}{2}\right) \right)$$

$$f(0) = 0, f(1) = 1$$

The third part of the paper is devoted to the study of the properties of the function  $f(x)$  defined by the equation

$$f(x) = \frac{1}{2} \left( f\left(\frac{x}{2}\right) + f\left(\frac{x+1}{2}\right) \right)$$



lo del ángulo " $2\gamma_4$ " que dos caras triangulares de este tipo contiguas por su base, del poliedro derivado, están en un mismo plano y forman un paralelogramo de lados iguales (rombo), este paralelogramo será pues un cuadrado.

Por lo anteriormente expuesto, llegamos a la conclusión de que "el poliedro derivado del tetraedro regular", ha de tener "seis caras", todas iguales, y en forma de "cuadrados", y por consiguiente dicha figura será "un hexaedro regular" (lo que demuestra lo expresado en el "Proceso gráfico").

Diagonal o lado "t" que se obtiene al unir los extremos de dos lados consecutivos del polígono de una cara del tetraedro dado

en este caso de cara triangular, será

$$t = l_4 = \frac{2\sqrt{6}}{3} a_4$$

Ángulo rectilíneo del diedro " $2\gamma_4$ " formado por dos caras laterales contiguas en las aristas de la pirámide recta

Se deduce de la fórmula general [7] obtenida anteriormente (ver consideraciones previas), sustituyendo en ella los valores particulares de este caso.

$$\text{sen } \gamma_4 = \frac{t q}{2 l_4 p} = \frac{\frac{2\sqrt{6}}{3} a_4 \times \frac{2\sqrt{3}}{3} a_4}{2 \times \frac{2\sqrt{6}}{3} a_4 \times \frac{\sqrt{6}}{3} a_4} = \frac{\sqrt{3}}{\sqrt{6}} = \sqrt{\frac{3}{6}} = \frac{1}{\sqrt{2}} =$$

The first of the series of lectures on the history of the United States was given by Mr. [Name] on the 1st of [Month] at the [Location]. The lecture was well attended and the speaker gave a very interesting and instructive account of the early history of the country.

The second lecture was given by Mr. [Name] on the 8th of [Month]. The subject was the history of the United States from 1789 to 1861. The speaker gave a very interesting and instructive account of the early history of the country.

The third lecture was given by Mr. [Name] on the 15th of [Month]. The subject was the history of the United States from 1861 to 1898. The speaker gave a very interesting and instructive account of the early history of the country.

The fourth lecture was given by Mr. [Name] on the 22nd of [Month]. The subject was the history of the United States from 1898 to the present. The speaker gave a very interesting and instructive account of the early history of the country.

The fifth lecture was given by Mr. [Name] on the 29th of [Month]. The subject was the history of the United States from the present to the future. The speaker gave a very interesting and instructive account of the early history of the country.

The sixth lecture was given by Mr. [Name] on the 5th of [Month]. The subject was the history of the United States from the future to the present. The speaker gave a very interesting and instructive account of the early history of the country.

$$= \frac{\sqrt{2}}{2}; \quad \text{y por lo tanto: } \gamma_4 = \frac{\pi}{4} = 45^\circ \quad \text{y} \quad 2\gamma_4 = \frac{\pi}{2} = 90^\circ$$

Desarrollo del cálculo anterior:

$$\boxed{\operatorname{sen} \gamma_4} = \frac{2\sqrt{6}}{3} a_4 \times \frac{2\sqrt{3}}{3} a_4 : \frac{2 \times 2\sqrt{6}}{3} a_4 \times \frac{\sqrt{6}}{3} a_4 = \frac{\sqrt{3}}{\sqrt{6}} = \sqrt{\frac{1}{2}} = \boxed{\frac{\sqrt{2}}{2}}$$

cuyo resultado nos confirma el ángulo recto que forman dos caras contiguas del exaedro regular.

Ángulo diedro " $\beta_4$ " formado por una cara lateral de la pirámide y su base

Se deduce de la fórmula general [8] (ver consideraciones previas) sustituyendo en ella los valores particulares de este caso.

$$\operatorname{sen} \beta_4 = \frac{a_4 - c_4}{b} = \frac{a_4 - \frac{1}{3} a_4}{\frac{\sqrt{6}}{3} a_4} = \frac{1 - \frac{1}{3}}{\frac{\sqrt{6}}{3}} = \frac{2}{3} : \frac{\sqrt{6}}{3} = \frac{\sqrt{6}}{3}$$

su valor numérico, expresado en grados sexagesimales, se obtiene:

$$\operatorname{sen} \beta_4 = \frac{\sqrt{6}}{3} = 0,81\ 64\ 96\ 6 \dots \quad \text{lg } \operatorname{sen} \beta_4 = \overline{1},911\ 95\ 44$$

$$\beta_4 = 54^\circ\ 44'\ 8,2''$$

$$\text{Comprobación: } \alpha_4 = \varphi_4 + \beta_4 = \frac{70^\circ\ 31'\ 43,6''}{2} + 54^\circ\ 44'\ 8,2'' = 90^\circ$$

(ver fórmula [11])



The first part of the paper is devoted to the study of the properties of the function  $f(x)$  defined by the equation

$$f(x) = \frac{1}{x} \int_0^x f(t) dt$$

and the second part is devoted to the study of the properties of the function  $f(x)$  defined by the equation

$$f(x) = \frac{1}{x} \int_0^x f(t) dt$$

and the third part is devoted to the study of the properties of the function  $f(x)$  defined by the equation

$$f(x) = \frac{1}{x} \int_0^x f(t) dt$$

and the fourth part is devoted to the study of the properties of the function  $f(x)$  defined by the equation

$$f(x) = \frac{1}{x} \int_0^x f(t) dt$$



cuyo resultado nos confirma el cálculo directo de  $\alpha_4$ , realizado anteriormente.

Radio " $b_2$ " de la esfera tangente a las aristas laterales de las pirámides rectas cuyas bases son caras del tetraedro regular dado.

Se deduce de la fórmula general [9] (ver consideraciones previas) sustituyendo en ella los valores particulares de este caso.

$$b_2 = \sqrt{(a_4)^2 - \frac{9^2}{4}} = \sqrt{(a_4)^2 - \left(\frac{2\sqrt{3}}{3} a_4\right)^2 : 4} = \frac{\sqrt{6}}{3} a_4$$

$$\begin{aligned} \text{Desarrollo del cálculo anterior: } \boxed{b_2} &= \sqrt{(a_4)^2 - \left(\frac{2\sqrt{3}}{3} a_4\right)^2 : 4} = \\ &= \sqrt{(a_4)^2 - \frac{12}{9} : 4 (a_4)^2} = \sqrt{1 - \frac{1}{3}} a_4 = \sqrt{\frac{2}{3}} a_4 = \sqrt{\frac{6}{9}} a_4 = \boxed{\frac{\sqrt{6}}{3} a_4} \end{aligned}$$

Radio " $c_1$ " de la esfera inscrita en el poliedro derivado.

Se deduce de la fórmula general [10] (ver consideraciones previas) sustituyendo en ella los valores particulares de este caso.

$$c_1 = b_1 \text{ con } \alpha_4 = \frac{\sqrt{3}}{3} a_4 \times \text{sen } \frac{\pi}{2} = \frac{\sqrt{3}}{3} a_4$$



Q.1. A particle of mass  $m$  is moving in a circular path of radius  $r$  with a constant speed  $v$ . Find the change in momentum of the particle when it completes one full revolution.

Sol. The initial momentum of the particle is  $\vec{p}_i = m\vec{v}$ . After one full revolution, the particle returns to its initial position with the same speed  $v$  but in the opposite direction. Therefore, the final momentum is  $\vec{p}_f = -m\vec{v}$ . The change in momentum is  $\Delta\vec{p} = \vec{p}_f - \vec{p}_i = -m\vec{v} - m\vec{v} = -2m\vec{v}$ .

Q.2. A particle of mass  $m$  is moving in a circular path of radius  $r$  with a constant speed  $v$ . Find the change in kinetic energy of the particle when it completes one full revolution.

Sol. The initial kinetic energy of the particle is  $K_i = \frac{1}{2}mv^2$ . After one full revolution, the particle returns to its initial position with the same speed  $v$ . Therefore, the final kinetic energy is  $K_f = \frac{1}{2}mv^2$ . The change in kinetic energy is  $\Delta K = K_f - K_i = \frac{1}{2}mv^2 - \frac{1}{2}mv^2 = 0$ .

Q.3. A particle of mass  $m$  is moving in a circular path of radius  $r$  with a constant speed  $v$ . Find the change in angular momentum of the particle when it completes one full revolution.

Sol. The initial angular momentum of the particle is  $\vec{L}_i = m\vec{r} \times \vec{v}$ . After one full revolution, the particle returns to its initial position with the same speed  $v$  but in the opposite direction. Therefore, the final angular momentum is  $\vec{L}_f = -m\vec{r} \times \vec{v}$ . The change in angular momentum is  $\Delta\vec{L} = \vec{L}_f - \vec{L}_i = -m\vec{r} \times \vec{v} - m\vec{r} \times \vec{v} = -2m\vec{r} \times \vec{v}$ .

Q.4. A particle of mass  $m$  is moving in a circular path of radius  $r$  with a constant speed  $v$ . Find the change in the magnitude of the angular momentum of the particle when it completes one full revolution.

Sol. The magnitude of the initial angular momentum is  $L_i = mrv$ . After one full revolution, the magnitude of the final angular momentum is  $L_f = mrv$ . The change in the magnitude of the angular momentum is  $\Delta L = L_f - L_i = mrv - mrv = 0$ .

Este mismo valor se puede deducir de la fórmula equivalente [10'], en la que

$$c_1 = b_2 \text{ sen } \gamma_4 = \frac{\sqrt{6}}{3} a_4 \times \frac{\sqrt{2}}{4} = \frac{\sqrt{3}}{3} a_4$$

Área lateral "S" del poliedro derivado

Se obtiene como suma de las áreas laterales de las cuatro pirámides rectas triangulares, cuyas caras son triángulos isósceles de base " $l_4$ " y altura " $p$ ", valores ya determinados.

$$S = 4 \times 3 \times \frac{l_4 p}{2} = 6 \times \frac{2\sqrt{6}}{3} a_4 \times \frac{\sqrt{6}}{3} a_4 = 8 (a_4)^2$$

Este valor debe ser coincidente al deducirlo de lo ya obtenido en la lám. 2, fórmulas 18 y 11, al ser  $a_4 = a_6$ , ya que

$$[S] = S_6 = 6 l_6^2 = 6 \times \left( \frac{2}{\sqrt{3}} a_6 \right)^2 = \frac{6 \times 4}{3} (a_6)^2 = 8 (a_6)^2 = \boxed{8 (a_4)^2}$$

como así sucede efectivamente.

(sigue en hoja 27)

Example 1. Find the area of a rectangle whose length is 10 cm and breadth is 5 cm.

Solution: We know that the area of a rectangle is given by the formula

$$\text{Area} = \text{Length} \times \text{Breadth}$$

Here, Length = 10 cm and Breadth = 5 cm. Substituting these values in the formula, we get

$$\text{Area} = 10 \times 5 = 50 \text{ cm}^2$$

∴ The area of the rectangle is 50 cm<sup>2</sup>.

Example 2. Find the area of a square whose side is 6 cm.

Solution: We know that the area of a square is given by the formula

$$\text{Area} = \text{Side} \times \text{Side}$$

Volumen V del poliedro derivado

Se obtiene como suma del volumen del tetraedro dado y de las cuatro pirámides de sus caras.

$$V = V_4 + 4 \times \frac{S_3 \times h}{3} \quad \text{siendo "S}_3\text{" el área de una cara del tetraedro y "h" la altura de la pirámide.}$$

Para obtener  $V_4$  en función de  $a_4$ , ver lám. 1, fórmulas 9 y 1, que nos da

$$V_4 = \frac{\sqrt{2}}{12} (a_4)^3 = \frac{\sqrt{2}}{12} \left( \frac{4}{\sqrt{6}} a_4 \right)^3 = \frac{8\sqrt{3}}{27} (a_4)^3$$

$$\text{Desarrollo del cálculo anterior: } \boxed{V_4} = \frac{\sqrt{2}}{12} \left( \frac{4}{\sqrt{6}} a_4 \right)^3 =$$

$$= \frac{\sqrt{2}}{12} \times \frac{4^3}{6\sqrt{6}} (a_4)^3 = \frac{64}{72} \sqrt{\frac{2}{6}} (a_4)^3 = \frac{8}{9} \sqrt{\frac{1}{3}} (a_4)^3 = \frac{8}{9\sqrt{3}} (a_4)^3 =$$

$$= \boxed{\frac{8\sqrt{3}}{27} (a_4)^3}$$

Por otra parte tendremos que

$$S_3 = \frac{\sqrt{3}}{4} (a_4)^2 = \frac{\sqrt{3}}{4} \times \left( \frac{2\sqrt{6}}{3} a_4 \right)^2 = \frac{2\sqrt{3}}{3} (a_4)^2$$

$$\text{Desarrollo del cálculo anterior: } \boxed{S_3} = \frac{\sqrt{3}}{4} \times \left( \frac{2\sqrt{6}}{3} a_4 \right)^2 =$$

$$= \frac{\sqrt{3}}{4} \times \frac{24}{9} (a_4)^2 = \boxed{\frac{2\sqrt{3}}{3} (a_4)^2}$$

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y también que (ver lám. 1, formulas 1 y 3)

$$h = a_4 - C_4 = a_4 - \frac{\sqrt{6}}{12} a_4 = a_4 - \frac{\sqrt{6}}{12} \times \frac{4}{\sqrt{6}} a_4 =$$

$$= a_4 - \frac{1}{3} a_4 = \frac{2}{3} a_4$$

y finalmente

$$V = V_4 + 4 \times \frac{S_3 \times h}{3} = \frac{8\sqrt{3}}{27} (a_4)^3 + \frac{4 \times \frac{2\sqrt{3}}{3} (a_4)^2 \times \frac{2}{3} a_4}{3} =$$

$$= \frac{8\sqrt{3}}{3} (a_4)^3$$

Desarrollo del cálculo anterior:

$$\boxed{V} = \frac{8\sqrt{3}}{27} (a_4)^3 + \frac{4 \times \frac{2\sqrt{3}}{3} (a_4)^2 \times \frac{2}{3} a_4}{3} = \left[ \frac{8\sqrt{3}}{27} + \frac{4 \times 2 \times 2 \times \sqrt{3}}{27} \right] (a_4)^3 =$$

$$= \frac{24\sqrt{3}}{27} (a_4)^3 = \boxed{\frac{8\sqrt{3}}{9} (a_4)^3}$$

Este valor debe ser coincidente al deducirlo de los ya obtenidos en la lám. 2, formulas 19 y 11, al ser  $a_4 = a_6$ , ya que

$$\boxed{V} = V_6 = (V_6)^3 = \left( \frac{2}{\sqrt{3}} a_6 \right)^3 = \frac{8}{3\sqrt{3}} (a_6)^3 = \frac{8\sqrt{3}}{9} (a_6)^3 = \boxed{\frac{8\sqrt{3}}{9} (a_4)^3}$$

como así sucede efectivamente

Q. 1. Find the value of  $\sin^{-1}(\sin \frac{\pi}{6})$ .

Sol. We know that  $\sin^{-1}(\sin x) = x$  if  $x$  lies in  $[-\frac{\pi}{2}, \frac{\pi}{2}]$ .

Here,  $\frac{\pi}{6}$  lies in  $[-\frac{\pi}{2}, \frac{\pi}{2}]$ .  
 $\therefore \sin^{-1}(\sin \frac{\pi}{6}) = \frac{\pi}{6}$

Q. 2. Find the value of  $\cos^{-1}(\cos \frac{5\pi}{6})$ .

Sol. We know that  $\cos^{-1}(\cos x) = x$  if  $x$  lies in  $[0, \pi]$ .

Here,  $\frac{5\pi}{6}$  lies in  $[0, \pi]$ .  
 $\therefore \cos^{-1}(\cos \frac{5\pi}{6}) = \frac{5\pi}{6}$

Q. 3. Find the value of  $\tan^{-1}(\tan \frac{2\pi}{3})$ .

Sol. We know that  $\tan^{-1}(\tan x) = x$  if  $x$  lies in  $(-\frac{\pi}{2}, \frac{\pi}{2})$ .

Here,  $\frac{2\pi}{3}$  does not lie in  $(-\frac{\pi}{2}, \frac{\pi}{2})$ .  
 $\therefore \tan^{-1}(\tan \frac{2\pi}{3}) = \frac{2\pi}{3} - \pi = -\frac{\pi}{3}$

Q. 4. Find the value of  $\sin^{-1}(\sin \frac{5\pi}{4})$ .

Sol. We know that  $\sin^{-1}(\sin x) = x$  if  $x$  lies in  $[-\frac{\pi}{2}, \frac{\pi}{2}]$ .

Here,  $\frac{5\pi}{4}$  does not lie in  $[-\frac{\pi}{2}, \frac{\pi}{2}]$ .  
 $\therefore \sin^{-1}(\sin \frac{5\pi}{4}) = \frac{5\pi}{4} - \pi = \frac{\pi}{4}$

En el cuadro sinóptico que damos a continuación, resumimos los resultados anteriores.

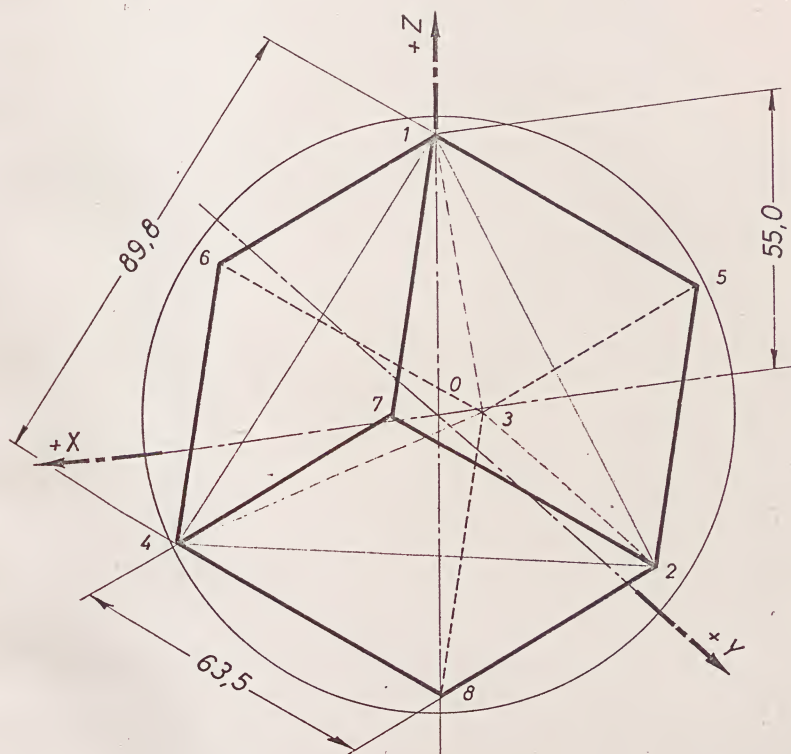
## CUADRO SINÓPTICO

Magnitud	Valor exacto	Valor decimal aproximado
<sup>236</sup> $l_4$	$\frac{2\sqrt{6}}{3} a_4$	1, 63 29 93... $a_4$
<sup>237</sup> $b_1$	$\frac{\sqrt{3}}{3} a_4$	0, 57 73 50... $a_4$
<sup>238</sup> $b_2$	$\frac{\sqrt{6}}{3} a_4$	0, 81 64 97... $a_4$
<sup>239</sup> $c_4$	$\frac{1}{3} a_4$	0, 33 33 33... $a_4$
<sup>240</sup> $c_1$	$\frac{\sqrt{3}}{3} a_4$	0, 57 73 50... $a_4$
<sup>241</sup> $d_4$	$\frac{2\sqrt{2}}{3} a_4$	0, 94 28 09... $a_4$
<sup>242</sup> $k_4$	$\frac{\sqrt{2}}{3} a_4$	0, 47 14 05... $a_4$
<sup>243</sup> $2\varphi_4$	$\text{sen } \varphi_4 = \frac{\sqrt{3}}{3}$	$\text{sen } \varphi_4 = 0, 57 73 50...$ $2\varphi_4 = 70^\circ 31' 43,6''$
<sup>244</sup> $2\alpha_4$	$\text{tg } \alpha_4 = \infty$	$2\varphi_4 = 180^\circ$
<sup>245</sup> $2\gamma_4$	$\text{sen } \gamma_4 = \frac{\sqrt{2}}{2}$	$\text{sen } \gamma_4 = 0, 70 71 07$ $2\gamma_4 = 90^\circ$
<sup>246</sup> $\beta_4$	$\text{sen } \beta_4 = \frac{\sqrt{6}}{3}$	$\text{sen } \beta_4 = 0, 81 64 97$ $\beta_4 = 54^\circ 44' 8,2''$
<sup>247</sup> $p$	$\frac{\sqrt{6}}{3} a_4$	0, 81 64 97... $a_4$
<sup>248</sup> $q$	$\frac{2\sqrt{3}}{3} a_4$	1, 15 47 01... $a_4$
<sup>249</sup> $t$	$\frac{2\sqrt{6}}{3} a_4$	1, 63 29 93... $a_4$
<sup>250</sup> $S$	$8 (a_4)^2$	8, 00 00 00... $(a_4)^2$
<sup>251</sup> $V$	$\frac{8\sqrt{3}}{9} (a_4)^3$	1, 53 96 01... $(a_4)^3$

FIGURA CORPÓREA

Se obtiene por acoplamiento de seis cuadrados de 63,5 mm de lado (ver lám. 2).

Year	Month	Day	Time	Place	Remarks
1901	Jan	1	10:00	St. Paul	Arrived
1901	Jan	2	10:00	St. Paul	Left
1901	Jan	3	10:00	St. Paul	Arrived
1901	Jan	4	10:00	St. Paul	Left
1901	Jan	5	10:00	St. Paul	Arrived
1901	Jan	6	10:00	St. Paul	Left
1901	Jan	7	10:00	St. Paul	Arrived
1901	Jan	8	10:00	St. Paul	Left
1901	Jan	9	10:00	St. Paul	Arrived
1901	Jan	10	10:00	St. Paul	Left
1901	Jan	11	10:00	St. Paul	Arrived
1901	Jan	12	10:00	St. Paul	Left
1901	Jan	13	10:00	St. Paul	Arrived
1901	Jan	14	10:00	St. Paul	Left
1901	Jan	15	10:00	St. Paul	Arrived
1901	Jan	16	10:00	St. Paul	Left
1901	Jan	17	10:00	St. Paul	Arrived
1901	Jan	18	10:00	St. Paul	Left
1901	Jan	19	10:00	St. Paul	Arrived
1901	Jan	20	10:00	St. Paul	Left
1901	Jan	21	10:00	St. Paul	Arrived
1901	Jan	22	10:00	St. Paul	Left
1901	Jan	23	10:00	St. Paul	Arrived
1901	Jan	24	10:00	St. Paul	Left
1901	Jan	25	10:00	St. Paul	Arrived
1901	Jan	26	10:00	St. Paul	Left
1901	Jan	27	10:00	St. Paul	Arrived
1901	Jan	28	10:00	St. Paul	Left
1901	Jan	29	10:00	St. Paul	Arrived
1901	Jan	30	10:00	St. Paul	Left
1901	Jan	31	10:00	St. Paul	Arrived



*Poliedro derivado del tetraedro regular*





Figure 1. A cube in perspective.







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ESTANTE.....

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NUMERO 7815.....

